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ON RAYLEIGH–TAYLOR INSTABILITY IN HEATED LIQUID–GAS AND LIQUID–VAPOR SYSTEMS

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The influence of the thermocapillary and phase change effects on the Rayleigh–Taylor instability of a heated two-layer system has been studied. For a system heated from the bottom the formulas have been derived for a critical gas or vapor layer thickness below which the Rayleigh–Taylor mechanism is affected by thermal effects. In a system with very thin gas layer the Rayleigh–Taylor instability is replaced by the thermocapillary one if the heat flux, transferred from the bottom wall, is sufficiently high. A motionless state of a very thin vapor layer can be stabilized by the phase change effect. Thermocapillary oscillatory instability has been found for large gas layer depths.

1. INTRODUCTION

The most susceptible wavelength of the Rayleigh–Taylor instability [1] is used in widely known models [2, 3] of the critical heat flux in boiling. In [2] it defines a distance between rising vapor jets, in [3] – a horizontal dimensions of vapor conglomerates located on a flat heater during the developed nucleate boiling. As the gravity level drops away, the Taylor wavelength becomes very large. For conditions of existing microgravity boiling experiments [4–6] it is several orders larger than the dimensions of heated plates [7]. As a consequence, the Rayleigh–Taylor instability is absent, and the model by Zuber [2] cannot longer be used to describe the transition from nucleate to film boiling observed in microgravity environments [6]. The model by Haramura and Katto [3] identifies the critical heat flux phenomenon with the dryout that occurs before the detachment of vapor conglomerates in a consequence of evaporation of a liquid macrolayer located beneath them. Recent experimental investigations show [8] that in the developed nucleate boiling with heat flux less than the maximum critical one, the macrolayer is repeatedly formed on a heater after every process of detachment. The consequence of events is shown in Figure 1 taken from the work [8]. Inside the macrolayer a number of vapor stems is formed by coalescence of small bubbles nucleating

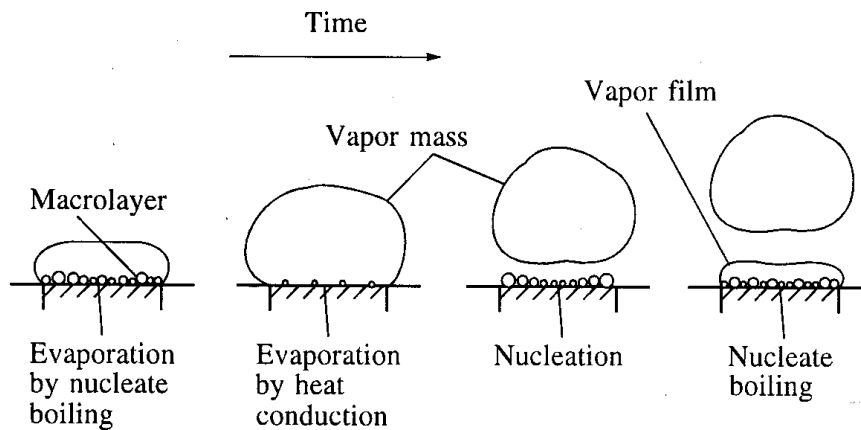


Figure 1. Sketch showing behavior of vapor mass and macrolayer.

on a heater after every process of detachment. A distance between vapor stems is much less than the lateral dimension of hovering above vapor mass. Obviously, that the location of vapor stems is not defined by Taylor wavelength.

In this work the Rayleigh–Taylor instability is studied in a heated systems at low Bond numbers, corresponding to low values of gravity acceleration or to small depths of vapor layer. The focus is on interactions between the Rayleigh–Taylor, the thermocapillary and phase change mechanisms. In order to isolate these three interfacial effects, we neglect in the present study the buoyancy forces arising due to the thermal expansion of fluids.

The Rayleigh–Taylor instability pertains to the isothermal static state of a horizontal fluid layer under a layer of a more dense fluid. If a constant heat flux is transferred across the interface of fluids, another cause of instability will be a surface tension gradient arising under the perturbations of the basic state (Smith, [9]). In a heated static system with the lower phase being a vapor one, the mentioned two mechanisms will be affected by the phase change occurring along the perturbed interface. On the basis of existing studies, it is difficult to predict whether the influence of the phase change is stabilizing or not.

In the work [10] Busse and Schubert obtained an interesting result on stability of the interface in a system with a heavy phase above. In general, they studied the Rayleigh–Bénard problem modifying it by involving an univariant phase transition. Their investigation was directed to applications in geophysical and astrophysical problems. Therefore, they restricted the consideration to the case with a small density jump across the phase-separating interface and equal viscosity and thermal diffusivity coefficients of the phases. In the particular case where the temperature gradient is nearly-adiabatic and the Rayleigh number is small, Busse and Schubert showed the stability of the basic state with the heavy phase above. This result is an example of the stabilizing influence of the phase change mechanism.

Another interesting result was obtained in a recent paper by Huang and Joseph [11], where the stability was examined for a motionless liquid-vapor system

confined between heated horizontal plates. The heat is transferred from the vapor to the liquid. Both phases are incompressible viscous fluids. The less dense vapor phase is above, and the Rayleigh–Taylor mechanism does not appear. Neglecting the variation of surface tension with temperature, the authors found the overstability and showed that it was due to the phase change mechanism. Also, they found that with very thick or very thin vapor layer, the motionless system was stable. However, they didn't explain why in these two exceptional cases the phase change was not able to give rise to the instability.

The important point discussed in the work [11] was concerned with the thermal boundary conditions at the interface. Two interfacial thermal conditions are required in the statement of the problem. The authors emphasize that these conditions are unknown. To motivate this they note, in particular, that when the pressure jump exists across the interface one cannot use the Clausius–Clapeyron relation for both phases together with the condition of temperature continuity at the interface. The validity of mentioned results have been verified for three cases: when the water and vapor are at saturation temperature corresponding to their own pressures and when the temperature is discontinuous and the vapor or the water is at saturation.

In the present work we assume thermal equilibrium, but we take into account the thermodynamic non-equilibrium effect at the perturbed interface. The well-known formula for the nonequilibrium phase change rate is the Hertz–Knudsen equation [12] derived from the kinetic theory of ideal gases. Because in our stability problem the vapor is assumed to be incompressible, to apply directly to this formula is incorrect. On the basis of the irreversible thermodynamics approach we derive the equation for the nonequilibrium mass flux to be used as the interfacial boundary condition in a statement of a stability problem. This approach allows to describe the interactions between the thermocapillary and phase change effects in terms of the phenomenological mass transport coefficient. Comparing the phenomenological mass transfer equation with the Hertz–Knudsen formula, we estimate the order of magnitude of the phenomenological coefficient through the accommodation coefficient. In our approach, the thermodynamic equilibrium condition for the interface is obtained as one that follows in the limiting case of infinite value of the phenomenological mass transport coefficient. With zero value of this coefficient the phase change does not occur, and the results are applied to the liquid–gas systems.

2. BASIC EQUATIONS

In this section the set of equations that govern the motion of a two-layer system will be written. The system consists of liquid layer $\zeta(x, y, t) < z < d$ in contact with its own vapor occupying the layer $-d_1 < z < \zeta(x, y, t)$ (Figure 2). At the rigid bottom $z = -d_1$ two kind of thermal conditions will be considered: a constant temperature or a constant heat flux. This set-up is used to investigate the influence of the phase change effect on the coupling between the Marangony–Bénard and

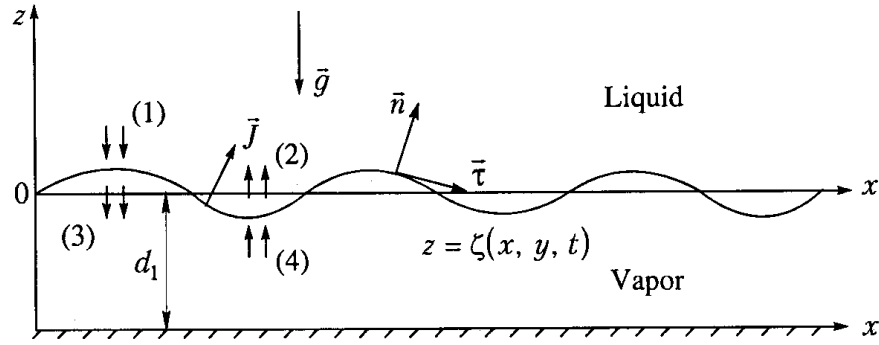


Figure 2. Physical configuration of liquid–vapor system. Four pair of vectors in the left side show the direction of motion induced by phase change in the absence of gravity: 1 – the liquid motion towards the crests, 2 – the liquid motion away from the troughs, 3 – the vapor motion away from the crests, 4 – the vapor motion towards the troughs.

the Rayleigh–Taylor instability mechanisms. In the basic state no motion exists, and the phases are separated by a flat interface $z = 0$ across which no net transfer of matter occurs.

The mass, momentum and energy conservation equations in each bulk are respectively

$$\nabla \cdot \mathbf{v} = 0 \quad (1)$$

$$\rho \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla p + \eta \nabla^2 \mathbf{v} + \rho \mathbf{g} \quad (2)$$

$$\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = k \nabla^2 T \quad \left(\nabla = \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \quad (3)$$

where p , T and $\mathbf{v} = (u, v, w)$ are the pressure, the temperature and the velocity. The constant parameters ρ , η and k are respectively the density, the dynamic viscosity and the heat diffusivity. The gravity acceleration vector is $\mathbf{g} = (0, 0, -g)$, where g is a positive constant. In the following, we will refer to the vapor phase by the subscript 1, for both physical parameters and the perturbed fields.

At $z = -d_1$ the no-slip and thermal conditions are as follows

$$\mathbf{v}_1 = 0, \quad T_1 = T_w \quad (\text{or } \lambda_1 \partial T_1 / \partial z = -q) \quad (4)$$

where for the first thermal condition T_w is the constant wall temperature, for the second one $-q$ is the constant value of the imposed heat flux. Note, that almost everywhere below the liquid depth is assumed to be infinite. With finite depth, at $z = d$ the imposed conditions are as for the perfectly conducting rigid plate.

At the liquid–vapor interface $z = \zeta(x, y, t)$ a continuity of the temperatures and of the tangential components of velocity is assumed

$$T = T_1, \quad (\mathbf{v} - \mathbf{v}_1) \cdot \boldsymbol{\tau} = 0 \quad (5)$$

Also, the mass, energy, the normal and tangential momentum balances are [13]:

$$\rho(\mathbf{v} - \mathbf{v}_\Sigma) \cdot \mathbf{n} = \rho_1(\mathbf{v}_1 - \mathbf{v}_\Sigma) \cdot \mathbf{n} \quad (6)$$

$$J \left\{ L + \frac{1}{2} [(\mathbf{v}_1 - \mathbf{v}_\Sigma) \cdot \mathbf{n}]^2 - \frac{1}{2} [(\mathbf{v} - \mathbf{v}_\Sigma) \cdot \mathbf{n}]^2 \right\} + \\ + \lambda \nabla T \cdot \mathbf{n} - \lambda_1 \nabla T_1 \cdot \mathbf{n} + 2\eta(\mathbf{D} \cdot \mathbf{n})(\mathbf{v} - \mathbf{v}_\Sigma) - 2\eta_1(\mathbf{D}_1 \cdot \mathbf{n})(\mathbf{v}_1 - \mathbf{v}_\Sigma) = 0 \quad (7)$$

$$J(\mathbf{v}_1 - \mathbf{v}) \cdot \mathbf{n} + p - p_1 - 2\mathbf{n} \cdot (\eta \mathbf{D} - \eta_1 \mathbf{D}_1) \cdot \mathbf{n} = \sigma \nabla \cdot \mathbf{n} \quad (8)$$

$$J(\mathbf{v}_1 - \mathbf{v}) \cdot \boldsymbol{\tau} + \mathbf{n}(\eta \mathbf{D} - \eta_1 \mathbf{D}_1) \cdot \boldsymbol{\tau} = -\nabla \sigma \cdot \boldsymbol{\tau} \quad (9)$$

Here L is the latent heat of vaporization, \mathbf{D} and \mathbf{D}_1 are the rate-of-deformation tensors, λ , λ_1 are the thermal conductivities, \mathbf{n} , $\boldsymbol{\tau}$ are the unit normal and tangential vectors (Figure 2), \mathbf{v}_Σ is the interface velocity, J is the normal component of the mass flux vector at the interface. By definition

$$J = \rho_1(\mathbf{v}_1 - \mathbf{v}_\Sigma) \cdot \mathbf{n} \quad (10)$$

The surface tension is assumed to vary linearly with the temperature T : $\sigma = \sigma_0 - \sigma_t(T - T_0)$. Here σ_0 is the surface tension at the temperature T_0 of the unperturbed interface $z = 0$; the temperature coefficient of surface tension σ_t is assumed to be a positive constant.

The system is not closed yet. One interfacial condition is missing that should characterize the interface thermodynamic state. The phenomenological expression

$$J = K(\mu_1 - \mu) \quad \text{at} \quad z = \zeta(x, y, t) \quad (11)$$

is postulated here in accordance with irreversible thermodynamics findings for particular case of no temperature jump at the interface [14–15] (see also [7]). It relates the normal mass flux J to the difference between the specific chemical potentials $\mu_1(T, p_1)$ and $\mu(T, p)$ of the vapor and of the liquid phases. The phenomenological coefficient K , depending [14] on the state of the vapor phase at the interface (the interfacial temperature T and the pressure p_1), is a positive function. Hence, the mass flux across the interface will occur from high to low chemical potential regions.

In the basic motionless state, at the interface $z = 0$ being at the pressure p_0 the material equilibrium condition [16]

$$\mu(T_0, p_0) = \mu_1(T_0, p_0) \quad (12)$$

holds. Relation (12) defines the point (T_0, p_0) at the phase coexistence curve [16]. The basic solution of the system (1) to (11) is given by

$$\mathbf{v}^0 = \mathbf{v}_1^0 = 0, \quad T^0 = -\frac{q}{\lambda}z + T_0, \quad T_1^0 = -\frac{q}{\lambda_1}z + T_0$$

$$p^0 = -\rho g z + p_0, \quad p_1^0 = -\rho_1 g z + p_0 \quad (13)$$

In the case where the temperature is kept constant at the rigid wall, the heat flux q in the solution (13) is equal to $\lambda_1(T_w - T_0)/d_1$.

In section 4, the problem on linear perturbations of solution (12), (13) will be formulated. But first, relation (11) will be linearized and examined in more details.

3. LINEARIZED EQUATION FOR THE MASS FLUX

Equation (11) defines the nonequilibrium mass flux across the interface. In the limiting case $K \rightarrow \infty$ from this equation follows (under the reasonable assumption that the mass flux is finite everywhere along the interface) that the chemical potentials are equal. This expresses the fact that the moving interface is in a state of quasi-equilibrium. In the case $K = 0$, the equality $J = 0$, following from Eq. (11), is the non-leak condition at the interface. Consequently, the problem with no phase change corresponds to this latter case. "No phase change" and "quasi-equilibrium" are two extreme cases of the phenomenological equation (11). In the next sections, we will see the profound difference in the stability analysis results in these two marginal cases.

To obtain a linearized form for Eq. (11) we introduce for every point of the interface $z = \zeta(x, y, t)$ with the temperature $T(x, y, t)$ a notion "quasistatic reference state" which is the motionless state of the liquid-vapor system with a flat interface at the temperature T ($T = T(x, y, t)$) and the saturation pressure $p_s(T)$. The difference of the chemical potentials vanishes at this reference state. Assuming that inside each phase local thermodynamic equilibrium holds near the interface $z = \zeta(x, y, t)$, we use the classical thermodynamics formulas [17]

$$\left(\frac{\partial \mu}{\partial p} \right)_{T, p_s(T)} = \frac{1}{\rho}, \quad \left(\frac{\partial \mu_1}{\partial p} \right)_{T, p_s(T)} = \frac{1}{\rho_1}$$

(valid for pure substances) and approximate the difference of the chemical potentials as follows

$$\mu_1 - \mu = \frac{p_1 - p_s(T)}{\rho_1} - \frac{p - p_s(T)}{\rho} \quad (14)$$

In a linear approximation the dependence $p_s(T)$ is defined by the formula

$$p_s(T) = p_0 + \frac{\rho \rho_1 L}{(\rho - \rho_1) T_0} (T - T_0) \quad (15)$$

obtained with the help of the Clausius–Clapeyron equation [16] for the slope of the phase coexistence curve. With the help of formulas (14), (15), Eq. (11) is easily reduced to the linearized form

$$J = -K_0 \frac{L}{T_0} \left[T - T_0 - \frac{T_0(\rho - \rho_1)}{\rho \rho_1 L} (p_1 - p_0) - \frac{T_0(p_1 - p)}{\rho L} \right] \quad \text{at } z = \zeta(x, t) \quad (16)$$

which will be used below in the linear stability analysis. In Eq. (16), the constant K_0 is the value of the phenomenological coefficient K at $T = T_0$, $p = p_0$. In the quasi-equilibrium limit, one obtains from (16) the expression for the interfacial temperature as follows:

$$T = T^{\text{eq}} = T_0 + \frac{T_0(\rho - \rho_1)}{\rho \rho_1 L} (p_1 - p_0) + \frac{T_0}{\rho L} (p_1 - p) \quad \text{at } z = \zeta(x, t) \quad (17)$$

The last term in Eq. (17) expresses the influence of the dynamic effects on the temperature T^{eq} of the perturbed interface which is, by assumption, at the quasi-equilibrium state. We shall consider the case when the density of vapor is much less than of liquid: $\rho_1 \ll \rho$. In this case the last term is small in comparison with the previous one, and formula (17) only slightly differs from the Clausius-Clapeyron relation for the vapor phase.

If one neglects a priori the last term in expression (14), then from the original equation (11) one gets the rate equation in the form wherein everywhere along the interface the mass flux J is proportional to the difference between the actual local vapor pressure p_1 at the interface and the equilibrium saturation pressure $p_s(T)$ corresponding to the local surface temperature. This form presents a formal analogy with the well-known Hertz-Knudsen equation [12]. Using this analogy we get the following rough estimation for the value K_0 through the accommodation coefficient β ($\beta \leq 1$) of the Hertz-Knudsen equation:

$$K_0 = \beta \rho_1 M^{1/2} / (2\pi R T_0)^{1/2} \quad (18)$$

Here: M is the molecular weight of vapor, R is the universal gas constant.

Equation (16) can be rewritten in the form:

$$J = -K_0 \frac{L}{T_0} (T - T^{\text{eq}}) \quad (19)$$

which shows that at any given point of the interface local nonequilibrium condensation ($J > 0$) or evaporation ($J < 0$) takes place following whether the actual temperature T is lower or greater than T^{eq} . Equation (19) shows also that the interface temperature T can be presented as a sum of its "equilibrium part" T^{eq} and a "nonequilibrium addition" $-JT_0/K_0L$. A more meaningful expression for the temperature T may be derived with the help of Eqs. (6), (7) and (8). If one expresses J from equation (7) and the pressure jump $p_1 - p$ from Eq. (8), then from Eq. (16), neglecting all nonlinear terms, one gets:

$$T = T_0 + T_0 \frac{\rho - \rho_1}{\rho \rho_1 L} (p_1 - p_0) - \frac{\sigma T_0}{\rho L} \nabla \cdot \mathbf{n} + \\ + \frac{2T_0}{\rho L} \mathbf{n} \cdot (\eta \mathbf{D} - \eta_1 \mathbf{D}_1) \cdot \mathbf{n} + T_0 \frac{(\lambda \nabla T \cdot \mathbf{n} - \lambda_1 \nabla T_1 \cdot \mathbf{n})}{K_0 L^2} \quad \text{at } z = \zeta(x, t)$$

This expression shows that the change of the surface temperature results from the pressure drop in the vapor phase, the interface curvature, jump of the normal viscous stresses and the jump of the heat flux at the interface.

4. LINEARIZED PERTURBATION EQUATIONS

We are considering here the governing equations (1)–(11), (16) in dimensionless form, choosing d_1 , k_1/d_1 , d_1^2/ν_1 , $\rho_1 k_1 \nu_1/d_1^2$ and $q d_1/\lambda_1$ as the length, velocity, time, pressure and temperature scales respectively. We formulate the problem on stability of the static state (12), (13) examining for simplicity 2-D perturbations. With the help of the formulas $u = -\Psi_z$, $w = \Psi_x$, we express the velocities of liquid and vapor through their stream functions, eliminate the pressures and linearize the system (1)–(10), (16) near the basic solution (13). Expanding the perturbations into normal modes by the Fourier developments

$$(\Psi, \Psi_1, T, T_1, \zeta) = (0, 0, T^0, T_1^0, 0) + \\ + (\psi(z), \psi_1(z), i\theta(z), i\theta_1(z), ia) \exp(nt + i\omega x)$$

we get the following eigenvalue problem.

In the liquid phase $0 < z < \infty$ ($D = d/dz$)

$$(D^2 - \omega^2)(D^2 - \omega^2 - \nu_* n) \psi = 0, \quad (D^2 - \omega^2 - n k_* \text{Pr}) \theta = -k_* \lambda_* \omega \psi \quad (20)$$

In the vapor layer $-1 < z < 0$

$$(D^2 - \omega^2)(D^2 - \omega^2 - n) \psi_1 = 0, \quad (D^2 - \omega^2 - n \text{Pr}) \theta_1 = -\omega \psi_1 \quad (21)$$

For the liquid phase the solutions satisfy the vanishing conditions at $z \rightarrow \infty$

$$\psi = D\psi = \theta = 0 \quad (22)$$

At $z = -1$ the no-slip and thermal conditions give respectively

$$\psi_1 = D\psi_1 = 0 \quad (23)$$

$$\theta_1 = 0 \quad (\text{or } D\theta_1 = 0) \quad (24)$$

At $z = 0$ one has the continuity conditions for the temperatures and tangential velocities

$$\theta_1 = \theta + (1 - \lambda_*)a, \quad D\psi = D\psi_1 \quad (25)$$

the mass balance and the jump balances of energy, normal and tangential stresses

$$(1 - \rho_*)n \text{Pr} a = \omega(\psi - \rho_*\psi_1) \quad (26)$$

$$Q(D\theta_1 - \lambda_*^{-1}D\theta) = \omega\psi_1 - n \text{Pr} a \quad (27)$$

$$\left[(1 - \rho_*^{-1})n + 3(1 - \eta_*^{-1})\omega^2 \right] D\psi + \eta_*^{-1}D^3\psi - D^3\psi_1 + \omega(\omega^2 - N_g)N_{\text{cr}}^{-1}a = 0 \quad (28)$$

$$s_e\tau_e^{-1}Q\omega(\theta - \lambda_*a) = \eta_*^{-1}(D^2 + \omega^2)\psi - (D^2 + \omega^2)\psi_1 \quad (29)$$

From Eq. (16) we find the temperature gradient along the interface, eliminate the gradients of the pressure and of the mass flux with the help of Eqs. (2), (10). Then we obtain at $z = 0$

$$Q\omega(\theta - \lambda_*a) = \tau_e D \left[D^2\psi_1 - v_*^{-1}D^2\psi - \omega^2(1 - v_*^{-1})\psi \right] - \tau_n\omega(\omega\psi_1 - n \text{Pr} a) \quad (30)$$

In the system (20)–(30) $\rho_* = \rho_1/\rho$, $v_* = v_1/v$ ($\eta_* = \eta_1/\eta$ is the dynamic viscosity ratio), $\lambda_* = \lambda_1/\lambda$, $k_* = k_1/k$, $\text{Pr} = v_1/k_1$ is the Prandtl number, $N_g = (\rho - \rho_1)gd_1^2/\sigma_0$ is the Bond number, $N_{\text{cr}} = \eta_1k_1/(\sigma_0d_1)$ is the crispation group. The parameter $Q = \lambda_1(T_w - T_0)/(\rho_1k_1L)$ can be called the modified Jakob number, $s_e = \sigma_t T_0/\rho_1 L d_1$ is the thermocapillary number. The dimensionless parameters $\tau_e = T_0\lambda_1 v_1/(\rho_1 L^2 d_1^2)$ and $\tau_n = T_0\lambda_1/(K_0 L^2 d_1)$ in (30) characterize respectively the variations along the interface of the temperature's "equilibrium part" and of its "nonequilibrium addition". In Eq. (29) the dimensionless group $s_e\tau_e^{-1}Q = \sigma_t q d_1^2/(\eta_1 k_1 \lambda_1)$ is the Marangoni number N_{Ma} for vapor phase.

If one multiply (30) by $s_e\tau_e^{-1}$, one obtains the explicit expression for the thermocapillary force $s_e\tau_e^{-1}Q\omega(\theta - \lambda_*a)$, which balances the jump of the tangential stresses in Eq. (29). From this expression one can see that in general case the thermocapillary effect is characterized by two dimensionless groups s_e and $s_n = \sigma_t T_0/(K_0 L v_1) = \tau_n s_e \tau_e^{-1}$. In the quasi-equilibrium limit one has $\tau_n = s_n = 0$, and the thermocapillary effect is characterized only by the parameter s_e . In the limit $\tau_n \rightarrow \infty$, Eq. (30) vanishes to the condition of no phase change. In this limit the Marangoni number can be used as the parameter which characterizes the instability. However, in the following, all results on stability will be presented with respect to the modified Jakob number Q . This is suitable for comparison of the marginal cases "no phase change" and "quasi-equilibrium".

5. GENERAL ANALYTICAL EXPRESSIONS FOR THE NON-OSCILLATORY MARGINAL CASE

We will first consider the case when the first condition (24) is imposed at the rigid wall. We set $n=0$ and obtain for Eqs. (20), (21) the solution satisfying the boundary conditions (22)–(29). For the functions ψ , ψ_1 , θ and for the amplitude of the surface deformation a we get

$$\psi = (Az + B)\exp(-\omega z), \quad \psi_1 = (L_1 z + U_1)\cosh \omega z - (N_1 z + V_1)\sinh \omega z \quad (31)$$

$$\theta = \left[C + \frac{k_* \lambda_*}{4\omega} A(z + \omega z^2) + \frac{k_* \lambda_*}{2} Bz \right] \exp(-\omega z) \quad (32)$$

$$a = 2\omega^2 N_{cr} (V_1 - v_*^{-1} U_1) / (\omega^2 - N_g) \quad (33)$$

where

$$A = \omega^{-1} \left[(\sinh^2 \omega - \omega^2) V_1 + (\omega + \sinh \omega \cosh \omega + \rho_* \omega^2) U_1 \right], \quad B = \rho_* U_1 \quad (34)$$

$$L_1 = \omega^{-1} \left[\sinh^2 \omega V_1 + (\omega + \sinh \omega \cosh \omega) U_1 \right] \quad (35)$$

$$N_1 = \omega^{-1} \left[(\omega - \sinh \omega \cosh \omega) V_1 - \cosh^2 \omega U_1 \right] \quad (36)$$

$$C = \frac{E_1(\omega) V_1 + E_2(\omega) U_1}{4(1 + \lambda_*^{-1} \tanh \omega)} + \frac{\tanh \omega U_1}{Q(1 + \lambda_*^{-1} \tanh \omega)} - \frac{(1 + \tanh \omega) a}{1 + \lambda_*^{-1} \tanh \omega} + \lambda_* a \quad (37)$$

with

$$E_1(\omega) = \omega^{-3} \left[\omega^3 - \sinh^2 \omega \tanh \omega + k_* (\sinh^2 \omega - \omega^2) \tanh \omega \right]$$

$$E_2(\omega) = \omega^{-3} \left[2\omega^2 - \omega(1 - \omega^2) \tanh \omega - \sinh^2 \omega + k_* (\omega + \sinh \omega \cosh \omega + 3\rho_* \omega^2) \tanh \omega \right]$$

To get the relationship between the amplitudes U_1 and V_1 we deduce from Eq. (29) the expression $\theta - \lambda_* a$, which is the total temperature perturbation along the surface, and introduce it into equality (30). Using formulas (31), (34)–(36) we obtain finally

$$V_1 = V(\omega) U_1 \quad (38)$$

where

$$V(\omega) = - \frac{E_3(\omega) + s_e E_4(\omega) - s_n \omega / 2}{E_5(\omega) + s_e E_6(\omega)} \quad (39)$$

with

$$E_3(\omega) = \omega^{-1} \left[\omega^2 + \cosh^2 \omega + \eta_*^{-1} (\omega + \sinh \omega \cosh \omega) \right]$$

$$E_4(\omega) = (1 - v_*^{-1})(\omega + \sinh \omega \cosh \omega) + \rho_* v_*^{-1} \omega^2$$

$$E_5(\omega) = \omega^{-1} [\sinh \omega \cosh \omega - \omega + \eta_*^{-1} (\sinh^2 \omega - \omega^2)]$$

$$E_6(\omega) = (1 - v_*^{-1}) \sinh^2 \omega + v_*^{-1} \omega^2$$

Note, that the mass flux through the interface is, by definition, proportional to the expression $\omega \psi_1(0)$, which is equal to ωU_1 , in accordance with formula (31). In the case of no phase change where $s_n \rightarrow \infty$, the function $V(\omega)$ tends to infinity. One has also $U_1 = 0$. Hence, Eq. (38) for the amplitude V_1 becomes undetermined. The particular case $s_n \rightarrow \infty$ will be examined in the next section. Here we assume s_n be finite. From Eq. (29) with the help of formulas (31)–(38) we get the neutral stability condition

$$\begin{aligned} Q \{ [E_1(\omega)V(\omega) + E_2(\omega)] - 8N_{cr}\omega^2(1 + \tanh \omega)[V(\omega) - v_*^{-1}] / (\omega^2 - N_g) \} = \\ = -4 \tanh \omega - 8\tau_e s_e^{-1} (1 + \lambda_*^{-1} \tanh \omega) [E_5(\omega)V(\omega) + E_3(\omega)] \end{aligned} \quad (40)$$

Identify now the physical origin of each term in relation (40). The second term in the right-hand side is proportional to the amplitude of the jump of tangential viscous stresses. The term linked to the temperature perturbation θ gives a contribution to the left-hand side (the first term) as well as to the right-hand side (the first term). The contribution to the right-hand side appears because, in contrast with the case of no phase change [9], the temperature perturbation at the interface (which is $\theta(0) = C$) depends (see (37)) on the temperature difference $T_w - T_0$ (on the number Q). The second term in the left-hand side reflects the contribution of the surface deformation to the total temperature perturbation.

In the case where the heat flux is kept constant at the rigid wall, the resolution follows the same line and the following neutral stability condition is obtained

$$\begin{aligned} Q \{ [H_1(\omega)V(\omega) + H_2(\omega)] - 8N_{cr}\omega^2(1 + \tanh \omega)[V(\omega) - v_*^{-1}] / (\omega^2 - N_g) \} = \\ = -4 - 8\tau_e s_e^{-1} (\lambda_*^{-1} + \tanh \omega) [E_5(\omega)V(\omega) + E_3(\omega)] \end{aligned} \quad (41)$$

where $V(\omega)$ is defined by (39) and

$$H_1(\omega) = \omega^{-3} [\omega(2 + \omega^2) \tanh \omega - \omega^2 - \sinh^2 \omega + k_* (\sinh^2 \omega - \omega^2)]$$

$$H_2(\omega) = \omega^{-3} [\omega(1 + \omega^2) + \omega^2 \tanh \omega - \sinh \omega \cosh \omega + k_* (\omega + \sinh \omega \cosh \omega + 3\rho_* \omega^2)]$$

For every liquid-vapor system the neutral stability curves can be plotted with the help of general formulas (40), (41) if all physical parameters of fluids are known. The oscillatory modes will be investigated on the bases of the general eigenvalue problem (20)–(30). We will perform all numerical calculations using the physical parameters of the system water with its vapor at $T_0 = 373$ K. These parameters are given below

$$\rho = 965 \text{ kg/m}^3, \quad \nu = 2.9 \cdot 10^{-7} \text{ m}^2/\text{s}, \quad k = 1.69 \cdot 10^{-7} \text{ m}^2/\text{s}, \quad \lambda = 0.68 \text{ J/(m s K)}$$

$$\rho_1 = 0.6 \text{ kg/m}^3, \quad \nu_1 = 2.08 \cdot 10^{-5} \text{ m}^2/\text{s}, \quad k_1 = 0.2 \cdot 10^{-4} \text{ m}^2/\text{s}, \quad \lambda_1 = 0.023 \text{ J/(m s K)}$$

$$L = 2.26 \cdot 10^6 \text{ J/kg}, \quad \sigma_0 = 0.06 \text{ N/m}, \quad \sigma_t = 0.2 \cdot 10^{-3} \text{ N/(m K)}$$

The value K_0 is unknown. For its estimation one can use formula (18). However, experimental values for the accommodation coefficient, reported over some decades by various authors, vary in a wide range. The reasons for existing discrepancy are not yet clear. In the light of existing data for pure substances the values of β can be assumed to be between 0.01 and 1. This corresponds to the values K_0 higher than $5.8 \cdot 10^{-6} \text{ kg}^2/(\text{m}^2 \text{s J})$ and less than $5.8 \cdot 10^{-4} \text{ kg}^2/(\text{m}^2 \text{s J})$. The interface pollution and the presence of the inert gases strongly influence on the rate of the mass transfer, leading to its essential decreasing (in [8] it has been found experimentally that for still water β varies from 0.001 to 0.02, depending on the purity of the water. For pure water system the values $K_0 < 5.8 \cdot 10^{-6} \text{ kg}^2/(\text{m}^2 \text{s J})$ are not, probably, physically realistic, but we will do also the calculations with $K_0 < 5.8 \cdot 10^{-6} \text{ kg}^2/(\text{m}^2 \text{s J})$ (working thus with some liquid-vapor system model) follow the interactions of the Rayleigh–Taylor, the thermocapillary and phase change mechanisms continuously as one changes K_0 from the limit of no phase change to the quasi-equilibrium one.

When in an isothermal system of two immiscible fluids, the lower phase has the lowest density, the static state is unstable due to the Rayleigh–Taylor instability. All infinitesimal perturbations whose wave number is smaller than the critical value $\omega_{RT} = N_g^{1/2} = [(\rho - \rho_1)g/\sigma]^{1/2} d_1$ are then exponentially amplified. On earth $g = g_0 = 9.81 \text{ m/s}^2$, and all perturbations with the wavelength larger than approximately 0.016 m (typical value for air below water) will be amplified. For earth conditions, with small values of d_1 the value of ω_{RT} is small. In microgravity conditions, as $g \rightarrow 0$, the critical wave number ω_{RT} tends towards zero as $g^{1/2}$. We thus have two different cases for which the Bond number is small and only small wave number perturbations are amplified by the Rayleigh–Taylor mechanism. In the following, it is examined how the thermocapillary and phase change effects influence on the stability of the system with respect to the long-wave perturbations.

6. THE NON-LEAK CASE

This is the limiting case $s_n \rightarrow \infty$. No mass transfer occurs from one phase to the other, and the coupling between the pure Marangoni-Bénard and the Rayleigh-Taylor instability mechanisms can be investigated. Equations (40), (41) can be particularized to the limiting case $s_n \rightarrow \infty$ by keeping the leading order terms (proportional to $V(\omega)$). They correspondingly reduce to

$$Q\left[E_1(\omega) - 8N_{cr}\omega^2(1 + \tanh \omega)/(\omega^2 - N_g)\right] = -8\tau_e s_e^{-1}(1 + \lambda_*^{-1} \tanh \omega)E_5(\omega) \quad (42)$$

for the case when a temperature is kept constant at the rigid plate and to

$$Q\left[H_1(\omega) - 8N_{cr}\omega^2(1 + \tanh \omega)/(\omega^2 - N_g)\right] = -8\tau_e s_e^{-1}(\lambda_*^{-1} + \tanh \omega)E_5(\omega) \quad (43)$$

for the case of constant heat flux condition. Note, that the first equation is equivalent to the neutral stability condition obtained earlier by Smith [9] for the system with an infinitely deep upper fluid.

We will now investigate the case of small wave numbers under the assumption that the physical parameters in the neutral stability conditions (42), (43) have the values typical of usual liquids and their vapors. Since typical values of η_*^{-1} and λ_*^{-1} are large, we assume that $\eta_*^{-1}\omega$ and $\lambda_*^{-1}\omega$ are of order unity. We also assume that $k_* > 1$. We then develop Eqs. (42), (43) in powers of ω and, keeping the leading order terms, get the following approximate expressions for $\omega \rightarrow 0$:

$$Q\left(\omega^2 - N_g - \frac{24N_{cr}}{k_*}\right) = -\frac{16\tau_e}{k_* s_e}(\omega^2 - N_g)\left(1 + \lambda_*^{-1}\omega\right)\left(1 + \frac{\eta_*^{-1}\omega}{2}\right) \quad (44)$$

$$Q\left(\omega^2 - N_g - \frac{24\omega N_{cr}}{k_*}\right) = -\frac{16\tau_e}{k_* \lambda_* s_e}(\omega^2 - N_g)\omega\left(1 + \frac{\eta_*^{-1}\omega}{2}\right) \quad (45)$$

It is worth noting here that formula (44) differs from the result obtained by Smith [9] in a long wave approximation. That is because, in contrast with this author's analysis performed for earth gravity conditions and for a system in which the semi-infinite gas domain lies above, we consider here the case of small Bond numbers N_g and of values of k_* higher than unity.

There exists a finite critical value of ω , for which the left-hand side of Eq. (44) (or of Eq. (45)) is equal to zero. This happens for $\omega_T = (N_{cr}/k_*)^{1/2}(24 + G_T)^{1/2}$ in the case of constant wall temperature, and for $\omega_q = N_{cr}k_*^{-1}\left(12 + (144 + G_q)^{1/2}\right)$ for a constant heat flux condition. The gravity parameters G_T and G_q are defined as follows

$$G_T = \frac{k_* N_g}{N_{cr}} = \frac{(\rho - \rho_1)gd_1^3}{\eta_l k}, \quad G_q = \frac{k_*^2 N_g}{N_{cr}^2} = \frac{(\rho - \rho_1)\sigma_0 gd_1^4}{\eta_l^2 k^2} \quad (46)$$

At the critical values of ω the left-hand side of (44) and of (45) changes sign. In the vicinity of points ω_T , ω_q the values of Q are infinitely large. The corresponding asymptote separates two marginal non-oscillatory instability curves. This is seen in Figures 3(a)–(c) plotted on the basis of the original equation (40). Whatever the kind of the boundary condition at the rigid wall, the numerical solution of the general eigenvalue problem (20)–(30) showed the existence of two real eigenvalues n_1 and n_2 (oscillatory instability will be discussed later). The first eigenvalue changes sign on the marginal non-oscillatory instability curve (curve 1, Figures 3(a)–(c)) lying in the domain $\omega < \omega_T$ (or $\omega < \omega_q$), the second one – on the curve 2, lying in the region $\omega > \omega_T$ (or $\omega > \omega_q$). Above the curve 1 the eigenvalue n_1 is positive, while n_2 is positive below the curve 2.

Two different mechanisms are acting to give an overall unstable situation. One is essentially linked to surface deformation. The other is due to convective mechanism. It is easy to compare the action of these mechanisms in the case $N_g = 0$, when the total temperature gradient at the interface $z = 0$ is the only one instability mechanism. We will do this for the case where the temperature is kept constant at the bottom wall. Here the temperature-gradient amplitude is $Q\omega[C(\omega) - \lambda_* a]$. Using formula (37) with $U_1 \equiv 0$, one finds that the amplitude is proportional to the expression $E_1(\omega)V_1 - 4(1 + \tanh \omega)a$. The first term expresses the change of a temperature gradient due to the convective mechanism while the second one is due to the surface deformation effect. From formula (33) with $N_g = 0$ one finds that for all $\omega \rightarrow 0$ the second term is equal to $-8N_{cr}V_1$. In the limit $\omega \rightarrow 0$ the first order approximation to the first term is $k_*\omega^2V_1/3$. One sees that if ω is small enough, the leading term is the second one. This shows that the instability above the curves 1 is linked to surface deformation effect. With increasing of ω the first term increases. For $\omega > \omega_T$, the absolute value of the first term becomes larger than of the second. Hence, the dominating mechanism of instability in the domain below the curve 2 is the convective mechanism. In the limit $\omega \rightarrow \infty$ both Eqs. (40), (41) reduce to the same expression $Q(1 - k_*) = 8\tau_e s_e^{-1}(1 + \lambda_*^{-1})(1 + \eta_*^{-1})\omega^2$. It is independent of the crispation group and of the Bond number. This reads that the influence of the surface deformation completely disappears for short wave perturbations.

For $N_g \neq 0$ every curve 1 intersects the ω -axis at the critical wave number ω_{RT} of pure Rayleigh–Taylor instability. (For the situation depicted in Figure 3(a) $\omega_{RT} = 1.98 \cdot 10^{-5}$ and the crossing point is not seen because it is very close to the coordinate origin). Numerical calculations of neutral stability curves show that with small N_g and N_{cr}/k_* , the asymptote positions are well predicted by the formulas for ω_T , ω_Q derived above in a long-wave approximation. This is seen in Figure 3(a) wherein the dot-dashed curves are plotted on the basis of Eq. (44). From Figure 3(a) to Figure 3(c) the gravity parameter G_T increases. Its value is

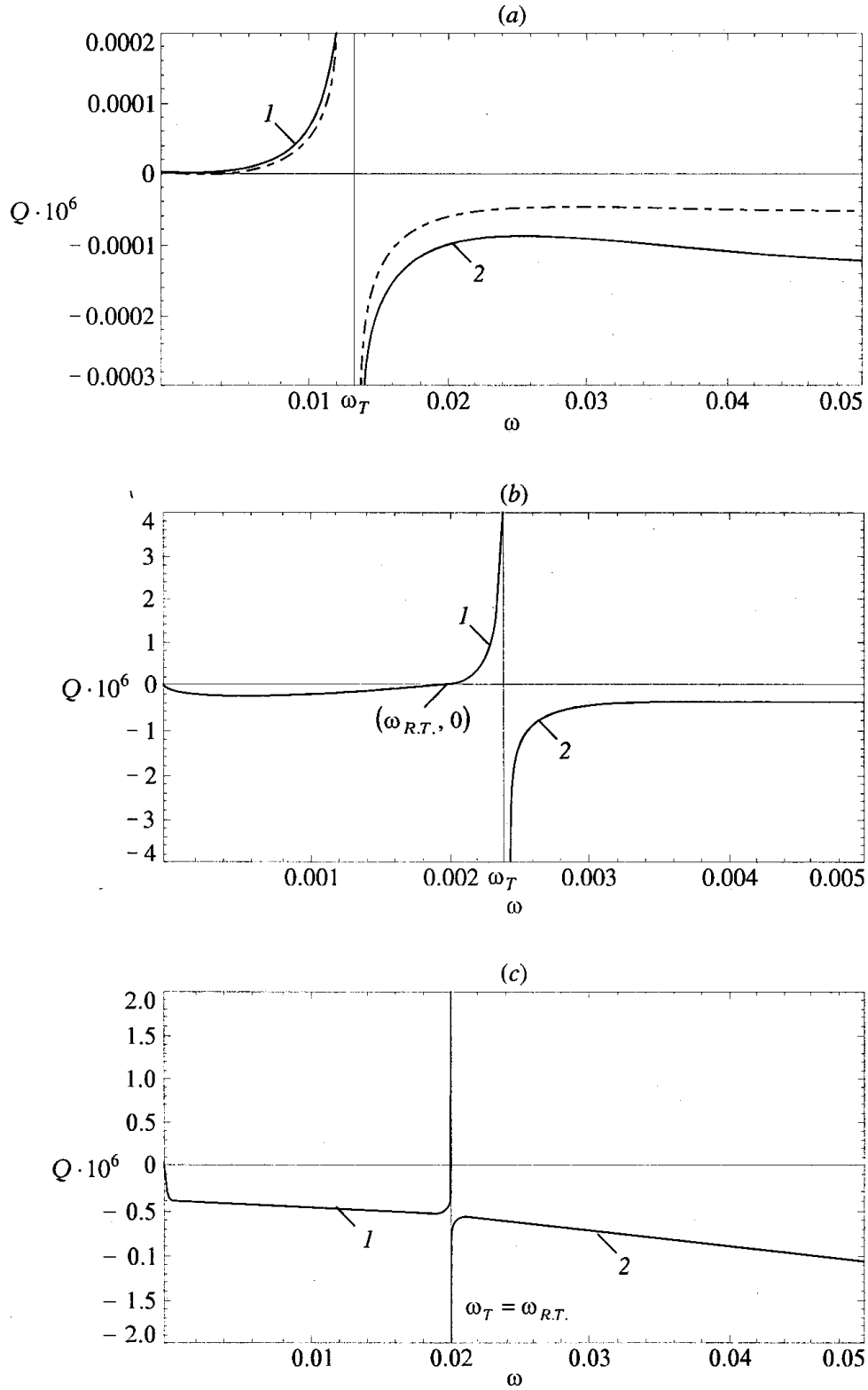


Figure 3. Neutral stability curves in the case of no phase change. Physical parameters of system are as for water with water vapor at $T_0 = 373$ K: (a) – $g = 9.81 \cdot 10^{-4}$ m/s², $d_1 = 5$ μ m; (b) – $g = 9.81 \cdot 10^{-4}$ m/s², $d_1 = 500$ μ m; (c) – $g = 9.81 \cdot 10^{-2}$ m/s², $d_1 = 500$ μ m. The dot-dashed curve represents the long wave approximation. Unstable zones are above the curve 1 and below the curve 2.

equal to $5.856 \cdot 10^{-5}$, 58.56 and 5856 in Figure 3(a), Figure 3(b) and Figure 3(c) respectively. For large positive Q the instability condition can be defined by the inequality $\omega < \omega_T$ ($\omega < \omega_q$). The influence of gravity of this instability criteria disappears if $G_T/24 \ll 1$ ($G_q/144 \ll 1$), so that in the formula for ω_T , (ω_q) the parameter G_T (G_q) can be neglected. This is a case in Figure 3(a) where with large Q the instability interval is much wider than the interval $(0, N_g^{1/2})$. Contrary, in Figure 3(c) $G_T/24 \gg 1$, positive Q , the interval of the wave number of growing perturbations is practically equal to the interval $(0, N_g^{1/2})$ of the Rayleigh–Taylor instability.

When Q is large and $G_T/24 \ll 1$ ($G_q/144 \ll 1$), the dimensional critical wavelength is $2\pi d_1/\omega_T$ ($2\pi d_1/\omega_q$) with $\omega_T \approx (24N_{cr}/k_*)^{1/2}$ ($\omega_q \approx 12N_{cr}/k_*$). It is much less than the critical Rayleigh–Taylor wavelength $2\pi d_1 N_g^{-1/2}$. One easily shows that the square of ratio between the actual critical wavelength and the critical Rayleigh–Taylor one is equal to the small parameter $G_T/24$ ($G_q/144$). The dimensional critical Rayleigh–Taylor wavelength is independent of fluid depths. For the thermocapillary instability the situation is contrary to that. Here the critical wavelength decreases with decreasing the vapor depth. Note finally, that at earth gravity for the water–air system at 288 K to the values $G_T = 24$ and $G_q = 144$ there correspond respectively the depths $d_1 = 21 \mu\text{m}$ and $d_1 = 1.4 \mu\text{m}$.

7. QUASI-EQUILIBRIUM LIMIT

To get the neutral stability conditions for the case $s_n = 0$ one has to replace in formulae (40), (41) the function $V(\omega)$ by

$$V^{\text{eq}}(\omega) = -\frac{E_3(\omega) + s_e E_4(\omega)}{E_5(\omega) + s_e E_6(\omega)} \quad (47)$$

For $\omega \rightarrow 0$, keeping the first order approximation to each term in Eqs. (40), (41) with $V(\omega) = V^{\text{eq}}(\omega)$, one correspondingly obtains the following approximate formulae

$$\begin{aligned} Q \left[\omega^2 + \frac{16N_{cr}}{k_* \eta_* (1 + 2s_e)} \omega + \frac{8N_{cr}}{k_* (1 + 2s_e)} - N_g \right] = \\ = -\frac{4(\omega^2 - N_g)}{3k_* (1 + 2s_e)} \left[(2 + \eta_*^{-1} + 3s_e) \omega^2 + 6\tau_e (1 + \lambda_*^{-1} \omega) (1 + 2\eta_*^{-1} \omega) \right] \end{aligned} \quad (48)$$

$$Q \left[\left(1 + \frac{16N_{cr}}{k_* \eta_* (1 + 2s_e)} \right) \omega^2 + \frac{8N_{cr}}{k_* (1 + 2s_e)} \omega - N_g \right] =$$

$$= -\frac{4(\omega^2 - N_g)\omega}{3k_*(1 + 2s_e)} \left[(2 + \eta_*^{-1} + 3s_e)\omega^2 + 6\tau_e\lambda_*^{-1}(1 + 2\eta_*^{-1}\omega) \right] \quad (49)$$

If $N_g = 0$, the left hand sides of Eqs. (48) and (49) have both the sign of the parameter Q , while both right hand sides are negative. Hence, in the domain of short wave numbers the neutral stability curves are lying in the half plane $Q < 0$. This result is opposite to that obtained in the previous section for the non-leak case in which for $N_g = 0$ the thermocapillary surface waves instability exists above the neutral stability curve lying in the upper half plane. The distinction is a consequence of the fact that for positive Q (heating from below) the phase change stabilizes all monotonical quasi-equilibrium perturbations. Here we are able to explain the stabilizing effect for an arbitrary wave number, but only by examining the neutral perturbations.

Consider first the ratio of the vapor normal velocity w_1 at $z = 0$ to the surface deformation ζ . For positive ω it may be written, after some manipulations, as

$$\frac{w_1|_{z=0}}{\zeta} = \frac{(\omega^2 - N_g)}{2N_{cr}\omega[V^{eq} - v_*^{-1}]} \quad (50)$$

with V^{eq} defined by (47). For $v_* \geq 1$ the function V^{eq} is negative since all functions E_i ($i = 3, 4, 5, 6$), defined in section 5, are positive. The ratio (50) is negative for $N_g = 0$. Hence, the phase change mechanism generates the motion with a structure shown in Figure 2: the liquid moves towards the crests and away from the troughs, and the vapor moves away from the crests and towards the troughs. At positive Q , in the basic state the liquid is more cold than the interface, while the vapor is more hot. Thus, owing to the motion generated by the phase change mechanism, the interface is cooled near the crests and heated near the troughs. Consider now the influence of interface deformation on the surface temperature. It should lead to the same effect since near the crests the surface is convex with respect to the cold liquid, while near the troughs – to the hot vapor side. In the examining here linear approximation this is seen from the expression for the amplitude of total temperature perturbation at $z = 0$, which is $Q(C - \lambda_*a)$ with $C - \lambda_*a$ defined by formula (37). Indeed, in this expression the term, connected with the surface deformation influence, is the product of Q and the last but one term in the expression (37). This term does not depend on N_g . Thus, with positive Q , for all nonzero ω its sign is opposite to the sign of the surface deformation amplitude a .

Let in a consequence of a perturbation, a part of the interface gets warm (cold) so that some volume of above liquid evaporates (condenses). Then at $N_g = 0$, for heating from below, both, the surface deformation effect and convection induced

by the phase change mechanism, cool (heat) this volume, leading to its condensation (evaporation). If $N_g \neq 0$ this is valid only for the perturbations with $\omega > N_g^{1/2}$ for which the right-hand side of equation (50) is negative. For the perturbations with $\omega < N_g^{1/2}$ the ratio (50) is positive; the redistribution in the pressure field due to the action of gravity forces changes the direction of the motion in comparison with that at pure weightlessness. If there were no phase change, this motion should promote an increase of the surface perturbation. However, due to the influence of surface deformation on the interface temperature the vapor may condense at the crests and the liquid may evaporate at the troughs. Thus at $\omega < N_g^{1/2}$, $Q > 0$ there exists the competition between the phase change stabilizing and gravity destabilizing mechanisms.

Notice, that in the case under consideration the thermocapillary effect is characterized by one dimensionless parameter, which is the number s_e . From Eqs. (48), (49) it is clearly seen that the influence of the thermocapillary effect on the long wave nonoscillatory instability is negligibly small provided that $s_e \ll 1$, or, equivalently, $d_1 \gg d_* = \sigma_t T_0 / (\rho_l L)$. The value d_* is very small. For water system at 373 K one has $d_* = 0.055 \mu\text{m}$.

Consider the neutral stability condition (48) with $s_e \ll 1$. If $G_T < 8$ then $N_g < 8N_{cr}/k_*$ and the left-hand side keeps the sign of Q for all positive ω . When this is the case, Eq. (48) defines one neutral stability curve $Q(\omega)$, along which $Q > 0$ for $\omega < \omega_{RT}$ and $Q < 0$ for $\omega > \omega_{RT}$. The example is depicted in Figure 4 plotted on the basis of Eq. (40) with $V(\omega)$ equal to $V^{eq}(\omega)$ for earth gravity condition, $d_1 = 10 \mu\text{m}$. The maximum value of Q , equal to $4 \cdot 10^{-8}$, is reached at $\omega = 2.4 \cdot 10^{-3}$. It is worth to note that the temperature difference $T_w - T_0$ corresponding to the maximum Q is extremely small ($4.5 \cdot 10^{-6}$ K). This means that the phase change inhibits the gravity waves at any heating from below.

For $G_T > 8$, Eq. (48) predicts the existence of two neutral stability curves divided by a vertical asymptote

$$\omega = \omega_T^{eq} = -8N_{cr}(k_*\eta_*)^{-1} + \left(N_g - 8N_{cr}k_*^{-1} + 64N_{cr}^2k_*^{-2}\eta_*^{-2}\right)^{1/2}$$

This situation is presented in Figure 5 for $g = 10^{-4}g_0$, $d_1 = 500 \mu\text{m}$. It is seen that the system is unstable for all values of Q . For $Q \rightarrow \infty$ the instability interval tends to $(0, \omega_T^{eq})$ where $\omega_T^{eq} = 0.0018$. The critical Rayleigh–Taylor wave number $\omega_{RT} = 0.002$ is only slightly larger than ω_T^{eq} . The instability region is hatched. It consists of two regions in every of which one eigenvalue n_1 or n_2 of the problem (20)–(30) is positive.

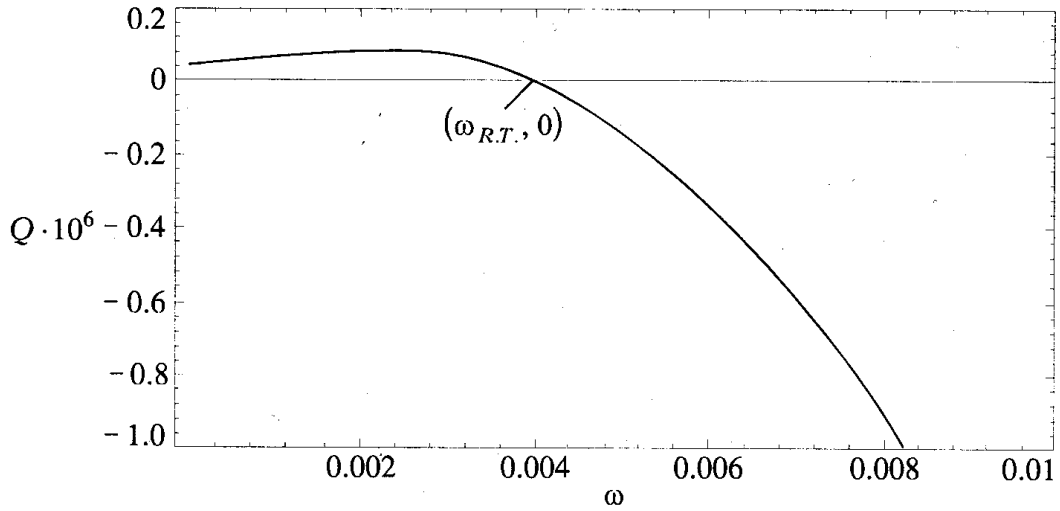


Figure 4. Neutral stability curve for the water system in the case of a quasi-equilibrium interface perturbations ($K_0 \rightarrow \infty$): $g = 9.81 \text{ m/s}^2$, $d_1 = 10 \text{ } \mu\text{m}$. The domain of instability is above the curve.

In Figure 6 the neutral stability curves are plotted for $d_1 = 100 \text{ } \mu\text{m}$ at different gravity levels. The curve 1 corresponds to $g = 0$. It is lying in the negative half plane. The static state is stable in the domain above this curve. For $g = 1.5 \cdot 10^{-3} g_0$ (curve 2) the phase-change mechanism stabilizes all perturbations if Q is higher than $2 \cdot 10^{-8}$ (the maximum value of Q along the curve 2). To the critical value $g^* = 1.8 \cdot 10^{-3} g_0$ ($G_T = 8$) there corresponds the curve 3 with the asymptote $\omega_T^{\text{eq}} = 0$. For the gravity acceleration $g = 3 \cdot 10^{-3} g_0$ two marginal stability curves (curves 4) exist. The domain of instability consists of two regions. One lies below the left curve 4, the second – between two curves 4. At this gravity level, for all positive Q the phase change cannot prevent the Rayleigh–Taylor instability.

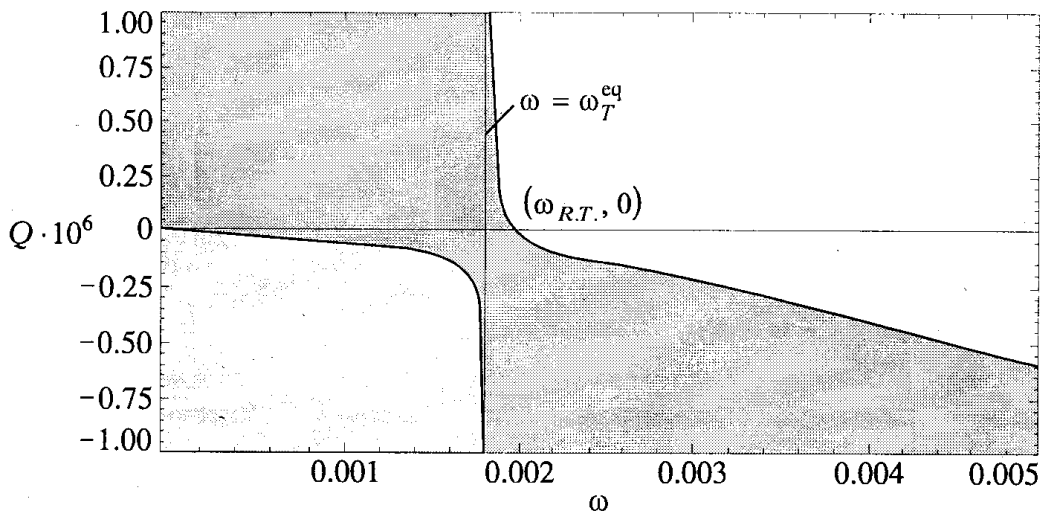


Figure 5. Neutral stability curve in the case $K_0 \rightarrow \infty$, $g = 9.81 \cdot 10^{-4} \text{ m/s}^2$, $d_1 = 500 \text{ } \mu\text{m}$. The state of no motion is unstable inside dashed regions.

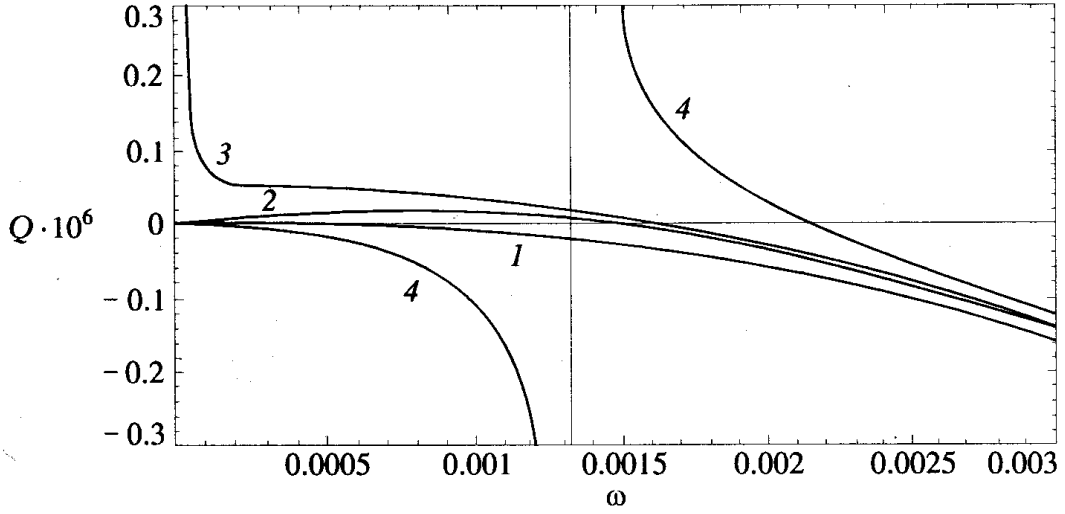


Figure 6. Neutral stability curves in the case $K_0 \rightarrow \infty$, $d_1 = 100 \text{ } \mu\text{m}$, and for various gravity levels (curve 1: $g = 0$, curve 2: $g = 1.5 \cdot 10^{-3} g_0$, curve 3: $g = g^* = 1.8 \cdot 10^{-3} g_0$, curve 4: $g = 3 \cdot 10^{-3} g_0 > g^*$, where g^* is the critical value above which an asymptote exists).

One essential point should be discussed with respect to the stabilizing influence of the phase change mechanism. The above analysis was performed for the case of infinitely deep liquid phase. With very long waves a stability of system will be affected by liquid depth. To investigate this effect we solved the stability problem (20)–(30) at $n = 0$, imposing the condition (22) at the boundary $z = d_*^{-1}$. For the obtained neutral stability condition the long wave approximation have been derived and investigated. Under the assumptions $d_* \ll 1$, $s_e \ll 1$, $\rho_* d_* \ll 1$, $\eta_*^{-1} d_* \ll 1$, $k_* d_*^{-2} \gg 1$, we found the stability condition for heating from below as follows: $G_T^1 = N_g^2 k_* / (N_{cr} d_*^2) < 160$. (Note, that the parameter G_T^1 coincides with G_T in particular case $d = \sigma_0^{1/2} / [(\rho - \rho_1)g]^{1/2}$). The obtained condition can be written in the form

$$\frac{(\rho - \rho_1)^2 g^2 d_1^3 d^2}{\sigma_0 \eta_1 k} < 160$$

which shows that, with given physical parameters, the stability takes place when the lower fluid has sufficiently small depth. Moreover, the stability condition is independent on the sign of the gravity acceleration g . It appears that this result confirms the result by Huang and Davis [11] mentioned above in the introduction.

Finally, consider the case of a constant heat flux at the rigid bottom where the long-wave approximation to the neutral stability condition is given by Eq. (49). Here the asymptote

$$\omega = \omega_q^{\text{eq}} = 8N_{cr} \left[-1 + \left(1 + G_q \left(1 + 16N_{cr} k_*^{-1} \eta_*^{-1} \right)^{1/2} \right) / 16 \right] / k_*$$

exists for all positive values of N_g . It is seen that for $16N_{cr}k_*^{-1}\eta_*^{-1}$ of order unity or less, with satisfying the inequality $G_q/16 \ll 1$ one has $\omega_q^{eq} \approx 0$. This predicts that for $G_q/16 \ll 1$, with sufficiently large positive values of Q , the Rayleigh-Taylor instability can be prevented by the stabilizing effect of the phase change.

8. THE CASE OF PURE WEIGHTLESSNESS

In this case oscillatory modes have been examined for positive Q by solving the general eigenvalue problem (20)–(30). The overstability caused by the thermocapillary mechanism has been found. For $K_0 = 0$ the marginal oscillatory curves are presented in Figure 7 for three vapor depths: 0.005 m, 0.01 m, 0.02 m. For every curve the domain of instability lies above. With decreasing the depth, the minimum critical value of Q , above which the instability exists, increases. For $d_1 = 0.005$ m it is equal to 0.064. For the water system model the corresponding critical temperature difference $T_w - T_0$ is equal to 72 K. With further decreasing of vapor depth the critical temperature difference increases rapidly. As a consequence, with very small depths, the oscillatory instability does not occur for physically realistic thermal conditions. In Figure 8 the frequency of overstable modes ($\omega_{osc} = \text{Im} n$, at $\text{Re} n = 0$) is plotted versus the wave number ω for the case $d_1 = 0.01$ m. To every curve there corresponds the marginal oscillatory curve $Q(\omega, K_0)$ along which the minimum of Q is reached at some critical value of ω . With increasing K_0 , the minimum value of Q increases. For $K_0 > 5.8 \cdot 10^{-6} \text{ kg}^2/(\text{m}^2 \text{sJ})$ ($\beta > 0.01$) the corresponding critical temperature differences are very large to be physically realistic. Hence, in the case under consideration the phase change does not cause the overstability. It is worth noting, that this

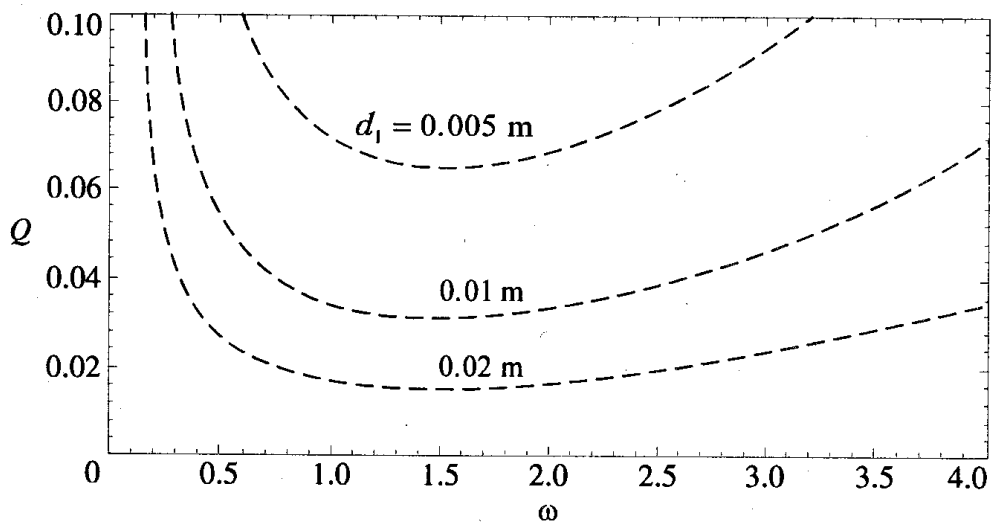


Figure 7. Marginal oscillatory curves for $K_0 = 0$, $g = 0$, and for three vapor depths – $d_1 = 0.005$ m, 0.01 m; 0.02 m. For every curve the domain of instability lies above.

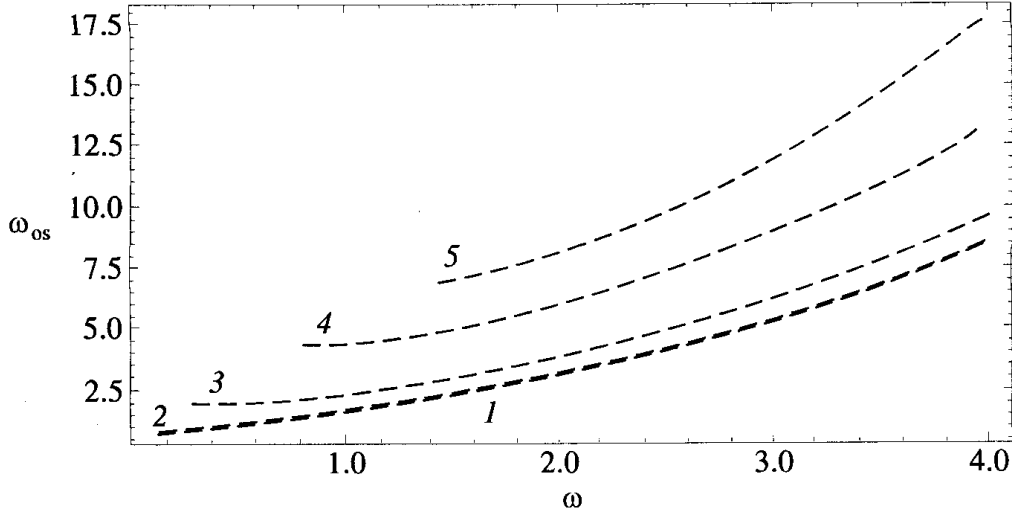


Figure 8. Frequency of overstable modes as a function of wave number. Depth $d_1 = 0.01$ m, $g = 0$. From the curve 1 to 5 the value of K_0 increases (curve 1: $K_0 = 0$, $K_0 = 10^{-10} \cdot 10^{i-2} \text{ kg}^2/(\text{m}^2 \text{ s J})$ for curve i , $i = 2, 3, 4, 5$).

result is not in contradiction with the result by Huang and Joseph [11] mentioned in section 1. In [11] a system was confined between two rigid plates. The oscillatory instability caused by the phase change was shown to exist with respect to perturbations of extremely long wave (10–100 m). In these author's study the influence of liquid depth on the behavior of perturbations was essentially important.

Consider now the long-wave approximation to the neutral stability conditions (40) and (41) assuming for simplicity that

$$\lambda_*^{-1}\omega \ll 1, \quad \eta_*^{-1}\omega \ll 1, \quad s_e \ll 1, \quad \rho_* \ll 1, \quad k_* \geq 1, \quad v_*^{-1} \leq 1 \quad (51)$$

Under the assumptions (51) we obtained from (39) the approximation $V(\omega) \sim -1.5(1 - s_n \omega^2)/\omega^3$ at $\omega \rightarrow 0$, then we reduce Eqs. (40), (41) respectively to the following

$$QP_T(\omega) = -\frac{16\omega^2\tau_e}{k_*s_e} \left[\left(1 + \frac{1}{\tau_n} \right) \omega^2 + 3 \frac{\tau_e}{\tau_n} \right] \quad (52)$$

$$QP_q(\omega) = -\frac{16\omega^2\tau_e}{\lambda_*k_*s_e} \left[\left(\omega + \frac{\lambda_*}{\tau_n} \right) \omega + 3 \frac{\tau_e}{\tau_n} \right] \quad (53)$$

Here $P_T(\omega)$, $P_q(\omega)$ are the polynomials of ω , defined as follows

$$P_T(\omega) = \omega^4 - 6N_{cr}k_*^{-1}(4 - \alpha)\omega^2 + 48N_{cr}^2k_*^{-2}\alpha \quad (54)$$

and

$$P_q(\omega) = \omega^3 - 24N_{cr}k_*^{-1}\omega^2 + 6N_{cr}k_*^{-1}\alpha\omega + 48N_{cr}^2k_*^{-2}\alpha \quad (55)$$

with

$$\alpha = \frac{k_*}{s_n N_{cr}} = \frac{\sigma_0 d_1 K_0 L}{\rho_1 k \sigma_t T_0} \quad (56)$$

Equations (52), (53) are presented in such a form where it is seen that, instead of Q , the Marangoni number for vapor phase $N_{Ma} = s_e \tau_e^{-1} Q$ can be used like the parameter which characterizes the thermocapillary mechanism of instability. But now it will be shown that, in a system heated from below, the condition of existence for the thermocapillary instability is formulated in terms of the dimensionless group α defined by (56).

The validity of the following assertion can be readily shown. If $0 < \alpha \leq 4/3$, the equation $P_T(\omega) = 0$ allows the real positive roots ω_{T1} , ω_{T2} , given by the formula

$$\omega_{T1,2} = \left(3N_{cr}k_*^{-1}\right)^{1/2} \left[4 - \alpha \mp (4/3 - \alpha)^{1/2} (12 - \alpha)^{1/2}\right]^{1/2}$$

The polynomial P_T is positive when $\omega < \omega_{T1}$ or $\omega > \omega_{T2}$. For $\omega_{T1} < \omega < \omega_{T2}$ it is negative. For $\alpha > 4/3$ the polynomial $P_T(\omega)$ is positive defined function for all values of ω .

Since the r.h.s. of equation (52) is negative, it follows from this assertion that, with negative Q , the static state is unstable for all values of α . Here the growing perturbations have the wave numbers $\omega < \omega_{T1}$ and $\omega > \omega_{T2}$; the mechanisms of instability are the phase change at the interface and the convection induced by the thermocapillary effect. For positive Q , the instability takes place only when α is small enough, namely, when $0 < \alpha < 4/3$. Long surface thermocapillary waves are the cause of instability. The wave numbers of dangerous perturbations belong to the interval $(\omega_{T1}, \omega_{T2})$ dislocated at a finite distance from the coordinate origin. Very long waves are stabilized by the phase change mechanism. In Figure 9(a) the neutral stability curves are plotted for the model system with the physical parameters as for water and water vapor, but for the value $K = 3 \cdot 10^{-12} \text{ kg}^2/\text{m}^2\text{sJ}$ that is very small to be physically realistic. The depth $d_1 = 100 \text{ } \mu\text{m}$. The instability takes place for three domains: above the curve 2, below the curve 1 and below the curve 3.

For the water system we calculated K_0 from Eq. (18). Then from the inequality $\alpha < 4/3$ we obtained the condition of existence for the thermocapillary instability as follows: $d_1 < d_1(\beta) = 1.3 \cdot 10^{-4} \beta^{-1} \text{ } \mu\text{m}$. In accordance with (51), $d_1 \gg 5.5 \times 10^{-2} \text{ } \mu\text{m}$ ($s_e \ll 1$). Therefore, this instability condition is valid only if $\beta \ll 2.3 \cdot 10^{-3}$. The more realistic instability condition $\beta < 1.2 \cdot 10^{-3}$ have been obtained for $d_1 = 5.5 \cdot 10^{-2} \text{ } \mu\text{m}$ ($s_e = 1$) from the long wave approximation to Eq. (40) derived for the case when in (51) the third assumption is not imposed.

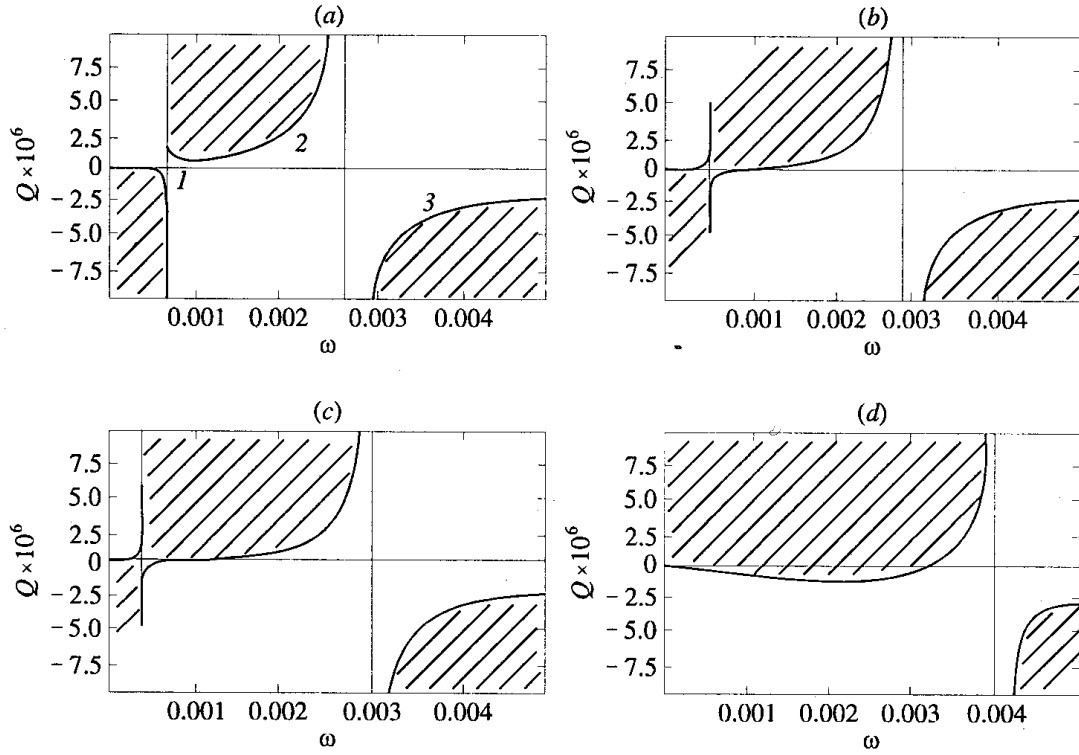


Figure 9. Neutral stability curve for the model system with physical parameters as for water with water vapor, $K_0 = 3 \cdot 10^{-12} \text{ kg}^2/(\text{m}^2 \text{ s J})$, $d_1 = 100 \text{ } \mu\text{m}$. Effect of increasing the gravity level: (a) – $g = 0$, (b) – $g = 5 \cdot 10^{-4} g_0$, (c) – $g = 10^{-3} g_0$, (d) – $g = 5 \cdot 10^{-3} g_0$. Domains of instability are dashed.

Note finally, that for a pure water system, the monotonical surface-wave thermocapillary instability is nonexistent in the case of a constant heat flux condition at the bottom. Here the approximation to the neutral stability condition is given by (53), (55). By direct calculations one can easily show that for all positive α the values $P_q(0)$, $P_q(24N_{\text{cr}}/k_*)$ are positive, while the value $P_q(12N_{\text{cr}}/k_*)$ is negative for $\alpha < 14.4N_{\text{cr}}/k_*$ and positive for $\alpha > 14.4N_{\text{cr}}/k_*$. Consequently, the polynomial $P_q(\omega)$ allows two roots in the interval $(0, 24N_{\text{cr}}/k_*)$ only when $\alpha < 14.4N_{\text{cr}}/k_*$. The last inequality is not valid with the values K_0 corresponding to the accommodation coefficient value between 0.01 and 1.

9. ON INTERACTIONS BETWEEN THE THERMOCAPILLARY, GRAVITY AND PHASE CHANGE MECHANISMS

Here the consideration is restricted to the case when the constant temperature is kept at the rigid wall. For small Bond numbers assuming conditions (51) be valid, we get the neutral stability condition with respect to the monotonical long-wave perturbations as follows

$$QP(\omega) = -\frac{16(\omega^2 - N_g)\tau_e}{k_*s_e} \left[\left(1 + \frac{1}{\tau_n} \right) \omega^2 + 3 \frac{\tau_e}{\tau_n} \right] \quad (57)$$

where

$$P(\omega) = \omega^4 - 6N_{cr}k_*^{-1}(4 - \alpha + G_T/6)\omega^2 + 48N_{cr}^2k_*^{-2}\alpha(1 - G_T/8) \quad (58)$$

The following assertion can be easily proven. If $0 < \alpha < G_T/2$ then the polynomial $P(\omega)$ allows two positive real roots for $0 < G_T < 8$ and the only one for $G_T > 8$. The neutral stability curves presented in Figures 9(b)–(d) correspond to the fixed value $\alpha = 5.2 \cdot 10^{-3}$ ($K_0 = 3 \cdot 10^{-12} \text{ kg}^2/(\text{m}^2 \text{ s J})$). The values of G_T are equal respectively to 2.23, 4.46 and 22.3. For the case shown in Figure 9(d), with positive Q , very long waves are not more stabilized by the phase change.

For pure water system the conditions of the above assertion are not fulfilled. Here $\alpha > 4 > G_T/2$, with positive Q , the behavior of long wave perturbations is determined by interaction of the gravity and the phase change mechanisms. Under the condition $G_T/24 \gg 1$, one can show that the polynomial (58) allows the only one positive root $\omega = N_g^{1/2}$. Hence, the Rayleigh–Taylor mechanism dominates if $G_T/24 \gg 1$. Further, by simple investigations one can show that for

$$0 < G_T < 8, \quad 3\alpha > 20 - G_T/2 - 16(1 - G_T/8)^{1/2}$$

the polynomial $P(\omega)$ is the positive defined function for all $\omega > 0$. It follows, that Eq. (57) defines the only one neutral stability curve. For $\omega > N_g^{1/2}$ the values of Q are negative along this curve. For $\omega < N_g^{1/2}$ the instability takes place only for extremely small values of Q . The situation is quite analogy with the case shown in Figure 4. If $G_T/8 \ll 1$, then the inequality for α reduces to the same stability condition as for pure weightlessness: $\alpha > 4/3$. Since for pure water system the inequality $\alpha > 4/3$ is valid, it follows that the inequality $G_T/8 \ll 1$ defines the condition under which the Rayleigh–Taylor mechanism disappears.

10. DISCUSSION AND CONCLUSIONS

Since the publication of Zuber's theory [2] several theoretical studies were performed (see [19] and refs. cited in [19]) to account for the influence of viscous effects on the Rayleigh–Taylor mechanism. The case with finite gas depth was considered in [20, 21]; the authors investigated the influence of contact angle on the critical heat flux value by examining a stability problem with the gas layer thickness equal to the bubble departure diameter. In the work [22] the phase change effect was taken into consideration, however, the vapor viscosity effect was ignored and infinite depth was assumed for vapor layer. The approach of the present work differs from the previous investigations in that simultaneously are examined the following three effects: finite (variable) vapor thickness, vapor viscosity and phase change. Moreover, the thermocapillary effect is involved, and two kind of thermal conditions on a wall are considered. Note, that the results of present paper (without

mathematical derivation) were earlier reported in [7] (for the case of a constant temperature condition) and in [23] (for a constant heat flux condition).

Discussing applications to boiling, one should stress that our model problem has "a shortcoming" in that the liquid in the basic state is always at the temperature less than the saturation temperature T_0 . In section 7, a subcooling of liquid phase was shown to be an important factor which stabilize the system and which even can prevent the Rayleigh–Taylor instability in the case of thin vapor layer. The situation with very thin vapor layer below the liquid occurs in the developed nucleate and transition boiling after the detachment of vapor conglomerates. In [8] this thin vapor layer is called "dry area". Let us present a description of boiling experiments on macrolayer formation made in [8]. "Just after a large bubble has detached from the surface of the disk heater, liquid rushing onto the surface comes into contact with the dry area and the heater surface is speedily covered with liquid in the form of a very thin liquid layer (macrolayer). The repeatedly formed macrolayer on the heater evaporates quickly and at CHF most of the heater surface is dried out. When the liquid rushing onto the surface comes into contact with dry area, a large amount of nucleation occurs simultaneously..." (see Figure 1). On the basis of the results obtained in our model problem with taking into account the mentioned "shortcoming" in the modeling of phase change effect, we have made in [7] an assumption that a simultaneous nucleation on a dry area can be connected with the thermocapillary instability of the interface between the liquid and the vapor residual layer. Such mode of instability has been described in section 6. Other modes of instability can be driven by the vapor recoil mechanism, by long-range molecular forces due to van der Waals attractions or by the Rayleigh–Bénard mechanism due to the dependence of liquid density on temperature. It seems, with instability conception it will be possible to predict such an important characteristic of macrolayer as the distance between the vapor stems. A more accurate modeling needs in more experimental information on thermal conditions above the "dry area".

For the case of heating from the bottom, important consequences of the performed analysis are the following. The length scale for the critical lower phase depth, below which the Rayleigh–Taylor mechanism is affected by thermal effects, is one and the same for the liquid-gas and liquid-vapor systems, but it depends on the kind of thermal condition imposed on a wall. When a constant heat flux is transferred from the wall, the scale is (see formulae (46))

$$\delta_q \sim \eta_l^{1/2} k^{1/2} [(\rho - \rho_1) g \sigma_0]^{-1/4} \quad (59)$$

while for the case of constant wall temperature it is defined by

$$\delta_T \sim \eta_l^{1/3} k^{1/3} [(\rho - \rho_1) g]^{-1/3} \quad (60)$$

Further, the obtained results can also be applied to the inverted systems in which the gas or vapor layer lies above the liquid. Here one has to change the sign

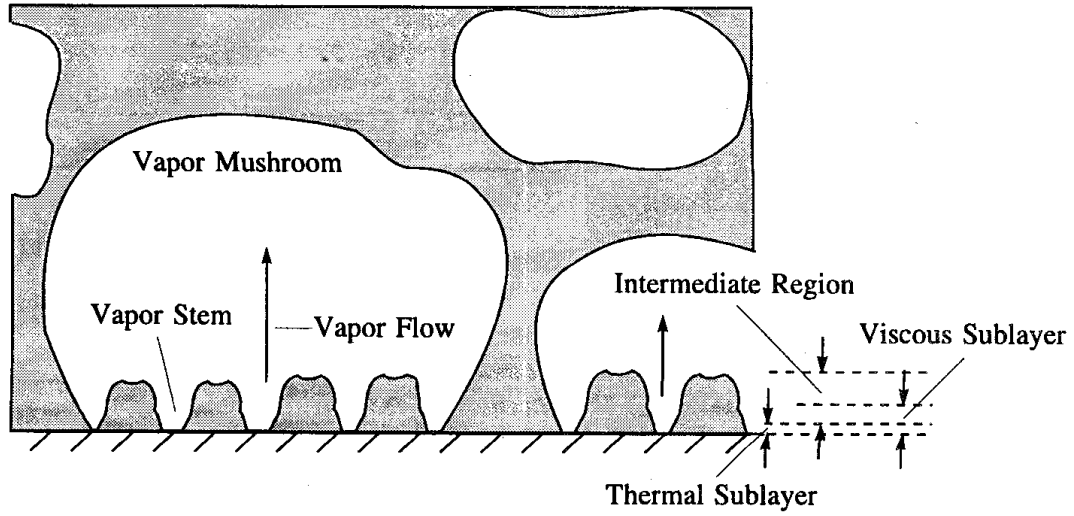


Figure 10. Sketch for the boiling process on a heated wall.

of the Bond number in general neutral stability conditions (40), (41) and, consequently, in their long wave approximations (44), (45) and (48), (49). The critical depth below which the thermal effects become dominating also exists and is defined by the same characteristic scales ((59), (60)).

It is believed that the scales (59), (60) can be used to characterize the thickness of thermal sublayer inside the liquid macrolayer beneath vapor mashrooms in the developed nucleate and transition boiling regimes (Figure 10). In this sublayer diffusivity effects are dominating; the heat is transferred by conduction from the wall through the liquid to the boundaries of vapor stems located inside the macrolayer. The region of macrolayer attached to the thermal layer can be called (see [24]) an intermediate layer, as it is still influenced by wall conditions. In this region the liquid evaporates at the boundaries of vapor stems, the evaporating mass is transferred along the stems to the above hovering vapor mass. Because the diameter of vapor stems increases with the approach to the above vapor conglomerate, the intermediate region is also influenced by the turbulent vapor flow dynamics away from the wall [24]. The existence of the two-phase laminar cell motion near the wall is due to that the dissipative viscosity effect is still important in the intermediate region. This effect is active inside the viscous sublayer of macrolayer (Figure 10). For scale reasons, the thickness of the viscous sublayer can be measured by $\Delta_q \sim \nu_1^{1/2} k^{-1/2} \delta_q$ or $\Delta_T \sim \nu_1^{1/3} k^{-1/3} \delta_T$ depending on the kind of thermal condition on a wall. One has correspondingly

$$\Delta_q \sim \rho_1^{1/2} \nu_1 [(\rho - \rho_1) g \sigma_0]^{-1/4}, \quad \Delta_T \sim \rho_1^{1/3} \nu_1^{2/3} [(\rho - \rho_1) g]^{-1/3}$$

Note, that the proposed consideration of the macrolayer differs from that in the critical heat flux theory by Haramura and Katto [3], where the inviscid model for macrolayer was suggested. Kutateladze [25] was the first to provide a theoretical formulation for the critical heat flux in boiling. In accordance with his assumption, the boiling crisis has the hydrodynamic nature and is connected with the

breach of stability of two-phase turbulent flow near the heated wall. We shall show now (see also [23]) that the formula by Kutateladze for the critical heat flux on an infinite horizontal plate can be obtained in the framework of the above consideration of macrolayer under the assumption that the critical heat flux is connected with the turbulization (chaotization) of the macrolayer and, thus, with a change of a turbulent flow structure in the region close to the wall. If one assumes that the turbulization occurs when the Reynolds number for vapor motion in vapor stems, defined as

$$Re = \frac{q \Delta q}{\rho_1 L v_1} \sim \frac{q}{\rho_1^{1/2} L [(\rho - \rho_1) \sigma_0 g]^{1/4}} = Re_{cr}$$

reaches the critical value then, under the assumption $Re_{cr} = \text{const}$, one obtains for the critical heat flux the formula by Kutateladze:

$$q_{cr} = \text{const } \rho_1^{1/2} L [(\rho - \rho_1) \sigma_0 g]^{1/4}$$

The influence of heater thickness and of its thermal properties on the critical heat flux can be explained by the fact that the thickness of viscous sublayer depends on these properties. Also, in general case, the critical Reynolds number Re_{cr} for vapor flow in intermediate region should depend on heater side properties (roughness), external flow conditions (working pressure, etc.), diameter (geometry) of vapor stems, the mass transport coefficient K . With strong dependence of the mass transport coefficient on the interface contamination one can explain a very essential difference [26] of the maximum heat flux values for tap and distilled water.

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NOMENCLATURE

a	amplitude of surface deformation, dimensionless
A, B, C	dimensionless amplitudes, eqs. (31), (32)
d, d_1	depth of liquid, vapor layer thickness, m
D, D_1	rate-of-deformation tensors
E_i	dimensionless functions, section 5 ($i = 1, 2, \dots, 6$)
g	gravity acceleration, m/s ²

g_0	earth gravity acceleration, m/s^2
H_1, H_2	dimensionless functions, section 5
J	normal mass flux across the interface, $\text{kg/m}^2\text{s}$
k, k_1	coefficients of heat diffusivity, m^2/s
K	mass transport coefficient, $\text{kg}^2/\text{m}^2\text{s J}$
L	latent heat of vaporization, J/kg
L_1, N_1	dimensionless amplitudes, eq. (31)
M	molecular weight of vapor, kg/mol
n	eigenvalue in the problem (20)–(30), dimensionless
$\mathbf{n}, \boldsymbol{\tau}$	unit normal and tangent vectors to the interface
p, p_1	pressures in liquid and vapor phases, N/m^2
P_T, P_q, P	polynomials (dimensionless), eqs. (54), (55), (58)
q	heat flux, $\text{J/m}^2 \text{ s}$
R	universal gas constant, J/(mol K)
T, T_1	temperatures of phases, K
t	time, s
U_1, V_1	dimensionless amplitudes, eq. (31)
$\mathbf{v} = (u, v, w)$	velocity vector, m/s
\mathbf{v}_Σ	interface velocity, m/s
V, V^{eq}	ratio of amplitude functions (dimensionless), eqs. (39), (47)
x, y, z	co-ordinates, m ; dimensionalized in section 4

Greek Symbols

β	accommodation coefficient, dimensionless
δ_T, δ_q	thickness of thermal sublayer, m
Δ_T, Δ_q	thickness of viscous sublayer, m
λ, λ_1	thermal conductivities, J/(m s K)
ρ, ρ_1	liquid and vapor densities, kg/m^3
μ, μ_1	specific chemical potentials, J/kg
ν, ν_1	kinematic viscosities, m^2/s
η, η_1	dynamic viscosities, N s/m^2
θ, θ_1	amplitudes of temperature perturbation, dimensionless
σ	surface tension coefficient, N/m
σ_t	temperature coefficient of surface tension, N/(m K)
Ω	wave number, m^{-1}
$\omega = \Omega d_1$	wave number, dimensionless
$\omega_{R.T.}$	critical wave number for pure Rayleigh–Taylor instability, dimensionless
ω_{T1}, ω_{T2}	positions of asymptote, dimensionless
ζ	interface level, m
$\Psi, \Psi_1(\psi, \psi_1)$	stream functions, m^2/s (dimensionless)

Subscripts

1	vapor
0	at unperturbed interface
o	at earth condition

T	under constant wall temperature
q	under constant heat flux on the wall
w	wall
e	due to variation of equilibrium part of surface temperature
n	due to variation of nonequilibrium part of surface temperature
$*$	ratio of liquid parameter to gas parameter
s	at saturation curve

Superscripts

0	basic state
eq	quasi-equilibrium limit

Criteria

$G_T = (\rho - \rho_1)gd_1^3/(\eta_1 k)$	gravity parameter
$G_q = (\rho - \rho_1)\sigma_0gd_1^4/(\eta_1^2 k^2)$	gravity parameter
$N_{cr} = \eta_1 k_1/(\sigma_0 d_1)$	crispation group
$N_g = (\rho - \rho_1)gd_1^2/\sigma_0$	Bond number
$N_{Ma} = \sigma_t q d_1^2/(\eta_1 k_1 \lambda_1)$	Marangoni number for vapor phase
$Pr = \nu_1/k_1$	Prandtl number
$Q = \lambda_1(T_w - T_0)/(\rho_1 k_1 L)$	Jakob modified number
$Re = q\delta_q/(\rho_1 L \nu_1)$	Reynolds number
$s_e = \sigma_t T_0/(\rho_1 L d_1)$	thermocapillary parameter
$s_n = \sigma_t T_0/(K_0 L^2 d_1)$	thermocapillary parameter
$\alpha = \sigma_0 d_1 K_0 L/(\rho_1 \sigma_t T_0)$	phase change influence on thermocapillarity number
$\tau_e = T_0 \lambda_1 \nu_1/(\rho_1 L^2 d_1^2)$	
$\tau_n = T_0 \lambda_1/(K_0 L^2 d_1)$	surface variation of temperature numbers

REFERENCES

1. Chandrasekhar, S., *Hydrodynamic and Hydromagnetic Stability*, Clarendon Press, Oxford, 1961.
2. Zuber, N., *Hydrodynamic Aspects of Boiling Heat Transfer*, A.E.C.U. Report № 4439, 1959.
3. Haramura, Y., Katto, Y., A New Hydrodynamic Model of Critical Heat Flux Widely Applicable to Both Pool and Forced Convection Boiling on Submerged Bodies in Saturated Liquids, *Int. J. Heat Mass Transfer*, **26**, P. 389–399, 1983.
4. Straub, J., Zell, M., Vogel, B., Boiling in Microgravity Conditions, *Proc. of the First Symposium on Fluids in Space*, Ajaccio, France, P. 269–297, 1991.
5. Grassi, W., Di Marco, D., Influence of Gravity and Electric Field Forces on Pool Boiling Heat Transfer, *Microgravity Quarterly*, **2**, P. 180–186, 1992.

6. Zell, M., Straub, J., Weinzierl, A., Nucleate Pool Boiling in Subcooled Liquid Under Microgravity. Results of Texus Experimental Investigations, Proceedings of the First European Symposium on Material Science in Microgravity, P. 327-333, Schloss Elmau, 1984.
7. Badratinova, L.G., Colinet, P., Hennenberg, M., Legros, J.C., Theoretical Models for Boiling at Microgravity, in Lecture Notes in Physics, L. Ratke, W. Walter and B. Feuerbacher, Eds., **464**, P. 361-370, Springer-Verlag, Berlin, 1996.
8. Kumada, T., Sakashita, H., Pool Boiling Heat Transfer-II. Thickness of Liquid Macrolayer Formed Beneath Vapor Masses, *Int. J. Heat Mass Transfer*, **38**, P. 979-987, 1995.
9. Smith, R.A., On Convective Instability Induced by Surface-Tension Gradients, *J. Fluid Mech.*, **24**, P. 401-414, 1966.
10. Busse, F.H., Schubert, G., Convection in a Fluid with Two Phases, *J. Fluid Mech.*, **46**, P. 801-812, 1971.
11. Huang, A., Joseph, D.D., Instability of the Equilibrium of a Liquid Below Its Vapour Between Horizontal Heated Plate, *J. Fluid Mech.*, **242**, P. 235-247, 1992.
12. Paul, B., Evaporation Coefficients, *ARS J.*, **32**, P. 9-14, 1962.
13. Delhaye, J.M., Jump Conditions and Entropy Sources in Two-Phase Systems. Local instant formation, *Int. J. Multiphase Flow*, **1**, P. 397-403, 1974.
14. Bornhorst, W.J., Hatsopolous, J.N., Analysis of a Liquid Vapour Phase Change by the Methods of Irreversible Thermodynamics, *Trans. ASME., J. Appl. Mech.*, P. 840-846, 1967.
15. De Groot, S.R., Mazur, P., Non-Equilibrium Thermodynamics, Dover Publications, Inc., New York, 1984.
16. Stephan, K., Heat Transfer in Condensation and Boiling, Springer-Verlag, Berlin, 1992.
17. Prigogine, I., Defay, R., Chemical Thermodynamics, Longman, London, 1973.
18. Hickman, K.C.D., Torpey, W.A., Evaporation of Resting Water, *Ind. & Eng. Chem.*, **46**, 1954.
19. Lienhard, J.H., Dhir, K., Extended Hydrodynamic Theory of the Peak and Minimum Pool Boiling Heat Fluxes, NASA CR-2270, 1973.
20. Kirichenko, Yu.A., Chernyakov, P.S., Some questions of Boiling Crisis in a Large Volume (in Russian), *Voprosy Gidrodinamiki i Teploobmena v Kriogennykh Sistemakh*, Tr. Fiz.-Tekhn. Inst. Nizk. Temp. Akad. Nauk SSSR, Yu.A. Kirichenko, I.S. Zhitomirskii, et al., Eds., P. 157-183, Kharkov, 1970.
21. Kirichenko, Yu.A., Chernyakov, P.S., To Definition of the First Critical Heat Flux on Planar Heaters (in Russian), *Inz.-Fiz. Zh.*, **20**, P. 982-987, 1971.
22. Hsieh, D.Y., Effects of Heat and Mass Transfer on Rayleigh-Taylor Instability, *Trans. ASME. J. Basic Engineering*, Ser. D, **94**, P. 156-160, 1972.
23. Badratinova, L.G., On Crisis of Transition from Nucleate to Film Boiling (in Russian), *Dinamika Splushnoi Sredy*, **111**, P. 15-20, 1996.
24. Dhir, V.K., Nucleate and Transition Boiling Heat Transfer under Pool and External Flow Conditions, Proceedings of the 9th Int. Heat Transfer Conf., Jerusalem, KN-9, P. 129-155, 1990.
25. Kutateladze, S.S., Hydrodynamic Theory for Change of Liquid Boiling Regime under Free Convection (in Russian), *Izv. Akad. Nauk SSSR, Ser. Tekh. Nauk*, **4**, P. 529-536, 1951.
26. Costello, C.P., Bock, C.O., Nichols, C.C., A Study of Induced Convective Effects on Pool Boiling Bornout, *Chem. Eng. Prog. Symp. Series*, **61**, P. 271-280, 1965.