

3 Thermal Convection of Visco-plastic Fluid

The rheological behavior of visco-plastic media is characterized by the yield stress τ_0 at which the system acquires fluidity. It is often described by the Bingham model which in the case of one-dimensional flow with the shear rate $\dot{\gamma}$ is the following:

$$\tau = \tau_0 \operatorname{sign}(\dot{\gamma}) + \mu_\infty \dot{\gamma} \qquad (2)$$

The actual media often have small but non-zero fluidity at small shear stress and only sharp change of the fluidity but not a jump takes place at $\tau \sim \tau_0$. Such behavior can be described with the help of Williamson model:

$$\tau = \{\tau_0/(\alpha + |\dot{\gamma}|) + \mu_\infty\}\dot{\gamma} \qquad (3)$$

At small α the model (3) is in fact the regularized Bingham model.

We have investigated the thermal buoyancy convection of visco-plastic fluid on the base of the Boussinesq equations supplied with the rheological relation (3) generalized for the three-dimensional case.

In the case of parallel g and ∇T the main results are close qualitatively to those which we obtained for the power-law pseudo-plastic fluid with the zero initial fluidity. The increase of plasticity leads to the increase of Ra_* i.e. to the stabilization of equilibrium.

The specific feature of visco-plastic behavior in the case of $\nabla T \perp g$ is that the convective motion is impossible at small Rayleigh numbers. Convection appears at the threshold value of the Rayleigh number when the stresses in the fluid connected with the temperature non-uniformity reach the yield stress. We have formulated two variational principles for the threshold conditions for the onset of convection in Bingham plastics. The minimization of obtained functionals made possible to find the estimates from above and from below for the ratio of threshold Rayleigh number to the yield stress and to determine the streamlines and the visco-plastic and quasi-rigid motions zones structure for the stationary convective flows near the threshold (fig. 4). Numerical investigation of this problem made with the help of finite-differences method for the Williamson model produced the close results for the threshold conditions for the onset of convection. The dependence $\Psi_m(Ra)$ obtained at $\tau_0 = 250$, $\alpha = 2.5$, $Pr = 10$, and $l = 2$ is presented in the fig. 5.

The results of the investigations show that the difference between Newtonian and non-Newtonian behavior is the most relevant at small Rayleigh numbers i.e. in the range of the parameters which are typical for the real microgravity conditions.

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Fig. 4. Streamlines and visco-plastic as well as quasi-rigid motion zones structure for the stationary convective flows near the threshold

Fig. 5. Extreme value of the streamfunction Ψ_m as function of the Rayleigh number (Ra) for $\tau_0 = 250$; $Pr = 10$ and $l = 2$

L. G. Badratinova, I. V. Belova, and N. A. Leontiev

Solidification of a Liquid Sphere under Weightlessness

As a sphere, initially fully melted and then cooled over the whole surface, transforms into a solid state, the stresses develop there due to spatial inhomogeneities of temperature field and admixture concentration. Considerable stresses may also develop due to the density variation upon crystallization. The aim of the present paper is to analyze a spherically symmetric process of crystallization of melted weightless sphere, cooled outwardly, and to investigate the stability of this process.

A mathematical model [1] is formulated, which describes the state of the material in a solid phase by the system of equations of isotropic quasi-static thermoelastoplasticity with two unknown boundaries. A convective motion in a liquid core is not taken into account. The heat equations for liquid and solid phases are conjugated via the Stefan type conditions, obtained in [1] from the general strong-discontinuity condition. This condition takes into account the work of surface forces in the energy balance at the crystallization front.

A spherically symmetric process was calculated numerically for different regimes of cooling of the external surface $r = R(t)$ [1–3]. It has been shown that as the crystallization front $r = S(t)$ moves towards the sphere center, pressure in the liquid phase increases, and this results in the interface velocity decrease. For metals and semiconducting materials, the time of full solidification increases by a factor of 2 to 4

Mail address: Dr. L. G. Badratinova, Lavrentyev Institute of Hydrodynamics, Lavrentyev prospect 15, Novosibirsk 630090, USSR.

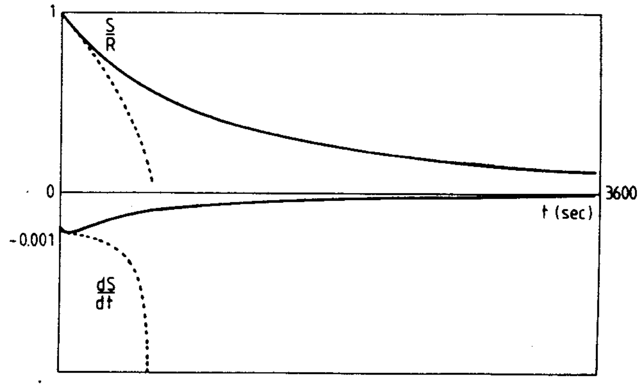

Fig. 1. Time dependence of a relative solid shell thickness $\gamma = (R - S)/R$ (upper half) and time dependence of the crystallization front velocity dS/dt for solidification of Si sphere; drawn line: thermoelastic model, dashed line: Stefan classical thermal model

as compared to the value calculated from the thermal Stefan problem. Fig. 1 illustrates the time dependence of a relative solid shell thickness $\gamma = (R - S)/R$ and the crystallization front velocity dS/dt for solidification of Si sphere. When $t = 0$ the radius of the sphere is $R_0 = 1$ cm, the temperature on the external surface is equal to the crystallization temperature T_0. The external surface is cooled with constant rate equal to 1 K/min. As is seen, in the thermoelastic model the time dependence of crystallization front velocity qualitatively varies as compared to the case of the Stefan classical thermal problem (dashed line). In the presence of alloying additives, the boundary layers appear near the moving crystallization front [2, 3]. The admixture storage takes place in a liquid phase thus decreasing the velocity of the front.

As soon as shear stresses reach the yield limit, there appears the region of plastic deformations in a solid phase. The stresses near the front are maximum. Therefore, with increasing t the interface $C(t)$ between the elastic and plastic regions moves from the crystallization front $S(t)$ towards

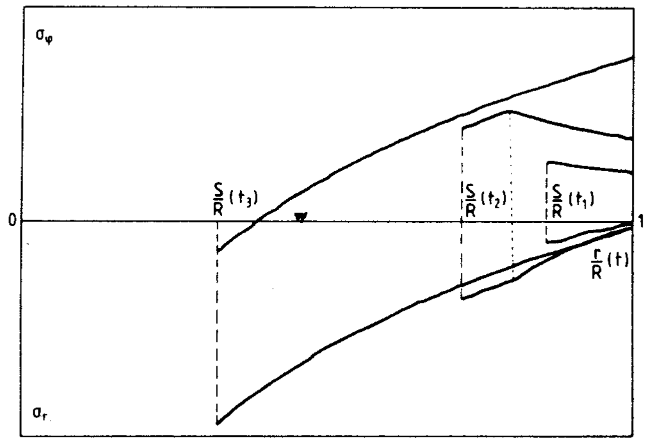

Fig. 2. Stress distribution (σ_r, σ_φ) along the radius of a solid shell; t_1: the solid phase is fully in an elastic state, t_2: some part of the solid phase is in an elastic state, the remaining part is in a plastic state, t_3: the solid phase is fully in a plastic state

the external boundary of the sphere $R(t)$. A typical stress distribution σ_r, σ_φ along the radius of a solid shell $S(t) < r < R(t)$ is shown in fig. 2 for three moments of time, t_1, t_2, and t_3. At the moment t_1 the solid phase is fully in an elastic state. When $t = t_2$, some part of the solid phase is in an elastic state and the remaining part is in a plastic one. At the moment t_3 the solid phase is fully in a plastic state.

The stability of the basic spherically symmetric solution is studied relative to quasi-stationary axisymmetric perturbations. The regime of cooling of the external surface remains to be spherical due to capillary forces. The main problem is to determine whether the process will be spherically symmetric up to a full solidification or a spherical form of the growing solid shell may be disturbed. If at some moment of time, there appear axisymmetric solutions along with a spherically symmetric one of the thermoelastoplastic problem, the disturbances may increase with time due to the increase in the pressure drop on the boundaries of the solid shell. In this case, the stability loss of the growing solid phase takes place.

The study of elastic stability shows that for a relative shell thickness $\gamma = 0.2$, there appear nontrivial axisymmetric fifth-order solutions, and for $\gamma = 0.36$ the solutions are axisymmetrical third-order ones. This result is valid for different specimens at the cooling rates being not higher than 1 K/s. The calculations were made for Si, Ge, Cu, Ag for a 1 cm initial radius of a sphere. Analogous results were obtained with increasing an initial radius of a specimen. If the cooling rate of a sphere is higher than 1 K/s, there develops the instability with respect to perturbations of the ninth-order (for $\gamma = 0.05$). The analogous results were obtained by the study of the stability of elastoplastic growing solid phase. At the moment of a full transformation into a plastic state, the instability develops relative to the perturbations whose order is higher than some value N_{min}. The latter depends on the thickness of a solid phase; its value is higher, the less the thickness of the solid shell. The latter result is in good agreement with the experimental data, presented in [4], and is likely to be indicative of the fact that the whole solid phase transforms into a plastic state almost as soon as the crystallization process starts.

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